

## Physiological and environmental regulation of stomatal conductance, photosynthesis and transpiration: a model that includes a laminar boundary layer\*

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### ABSTRACT

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This paper presents a system of models for the simulation of gas and energy exchange of a leaf of a C<sub>3</sub> plant in free air. The physiological processes are simulated by sub-models that: (a) give net photosynthesis ( $A_n$ ) as a function of environmental and leaf parameters and stomatal conductance ( $g_s$ ); (b) give  $g_s$  as a function of the concentration of CO<sub>2</sub> and H<sub>2</sub>O in air at the leaf surface and the current rate of photosynthesis of the leaf. An energy balance and mass transport sub-model is used to couple the physiological processes through a variable boundary layer to the ambient environment.

The models are based on theoretical and empirical analysis of  $g_s$  and  $A_n$  measured at the leaf level, and tests with intact attached leaves of soybeans show very good agreement between predicted and measured responses of  $g_s$  and  $A_n$  over a wide range of leaf temperatures (20–35°C), CO<sub>2</sub> concentrations (10–90 Pa), air to leaf water vapor deficits (0.5–3.7 kPa) and light intensities (100–2000  $\mu\text{mol m}^{-2} \text{s}^{-1}$ ).

The combined models were used to simulate the responses of latent heat flux ( $\lambda E$ ) and  $g_s$  for a soybean canopy for the course of an idealized summer day, using the 'big-leaf' approximation. Appropriate data are not yet available to provide a rigorous test of these simulations, but the response patterns are similar to field observations. These simulations show a pronounced midday depression of  $\lambda E$  and  $g_s$  at low or high values of boundary-layer conductance. Deterioration of plant water relations during midday has often been invoked to explain this common natural phenomenon, but the present models do not consider this possibility. Analysis of the model indicates that the simulated midday depression is, in part, the result of positive feedback mediated by the boundary layer. For example, a change in  $g_s$  affects  $A_n$  and  $\lambda E$ . As a consequence, the temperature, humidity and CO<sub>2</sub> concentration of the air in the proximity of the stomata (e.g. the air at the leaf surface) change and these, in turn, affect  $g_s$ . The simulations illustrate the possible significance of the boundary layer in mediating feedback loops which affect the regulation of stomatal conductance and  $\lambda E$ . The simulations also examine

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the significance of changing the response properties of the photosynthetic component of the model by changing leaf protein content or the CO<sub>2</sub> concentration of the atmosphere.

## INTRODUCTION

Transpiration and latent heat flux ( $\lambda E$ ) from vegetated surfaces are dependent on a number of interacting environmental and biological processes. One of the key elements of this system, the diffusive conductance of leaf surfaces to water vapor, may vary over a wide dynamic range and is controlled by physiological mechanisms. Most of the transpired water vapor, except when plant surfaces are wet, must pass through the stomata penetrating the leaf epidermis. It is generally accepted that these structures function to restrict water loss from the photosynthetic cells of the leaf mesophyll, yet they must also permit CO<sub>2</sub> to diffuse into the leaf to support photosynthesis. Complex physiological mechanisms adjust the opening of stomata in response to changes in environmental conditions, thus affecting the aggregate stomatal conductance of leaves ( $g_s$ ) and the canopy. Stomatal conductance is sensitive to the intensity of short-wave radiation ( $R_s$ ), humidity of the air ( $e_a$ ), CO<sub>2</sub> partial pressure of the air ( $c_a$ ), leaf temperature ( $T_l$ ), drought stress and time of day, as well as to physiological conditions in the plant (Jarvis, 1976; Stewart, 1988; Jones and Higgs, 1989). Some aspects of the stomatal mechanism, e.g. the turgor-dependent movements that open or close the pore, are fairly well understood (see Raschke, 1979; MacRobbie, 1987), but the ion transport processes and the sensory system that control these are still a puzzle.

Models that attempt to predict stomatal conductance directly from factors of the environment, while making no attempt to address the fundamental underlying mechanisms, are presently the approach of choice in constructing models of transpiration and energy balance of land surface. Of the models available, some have been obtained by correlation analysis of estimates of canopy conductance (estimated by inversion of the Penman–Monteith equation) with certain environmental variables such as solar radiation and humidity (Lindroth and Halldin, 1986; Stewart, 1988). Other models are based on direct studies of the stomatal conductance of leaves, either in controlled or natural environments (Jarvis, 1976; Avissar et al., 1985; Jones and Higgs, 1989). Some canopy models (Choudury and Monteith, 1986; Stewart, 1988; Gash et al., 1989) and global models (Sato et al., 1989) have included stomatal response functions. These studies have shown that physiologically mediated feedbacks affecting stomatal conductance may have strong influences on surface energy balance, and indicate the importance of having accurate models of this key process.

A general problem with correlative models of the type presently in use is that these tend to be specific to particular vegetation–climate systems (e.g.

Gash et al., 1989; Jones and Higgs, 1989). Parameters, such as the maximum stomatal conductance, or the sensitivity of  $g_s$  to the air to leaf vapor pressure deficit, must be readjusted (by empirical correlation) to fit a particular vegetation type or prevailing condition. A truly mechanistic model might be expected to provide a more general basis to predict stomatal responses, but such a model is not currently plausible.

In this paper, we will draw upon considerable body of work that has addressed the role of stomatal conductance in regulating the balance between transpiration and net uptake of  $\text{CO}_2$  in photosynthesis. This work considers functional and theoretical constraints that have presumably been major factors in the evolution of the regulatory mechanisms of stomata and, in our opinion more general and realistic models of  $g_s$  can be developed from proper consideration of these constraints. Nevertheless, the model we present is still largely empirical. Some parts of it, particularly those dealing with photosynthetic  $\text{CO}_2$  uptake, are based on fundamental properties of the biochemical mechanisms.

The theoretical treatment begins with the premise that stomata serve the conflicting roles of permitting  $\text{CO}_2$  to diffuse into the leaf to support photosynthesis and restricting the diffusion of water vapor out of the leaf. To succeed fully in one role must necessarily lead to total failure in the other role, and it has been argued that the regulatory system must strike an appropriate compromise between permitting photosynthesis and restricting water loss (Cowan, 1977; Raschke, 1979). Cowan and Farquhar (1977) proposed that the regulation of  $g_s$  over an interval of time would be optimal if no other response trajectory could have led to a smaller total water loss for a given carbon gain (or a larger carbon gain for a given water loss) integrated over that interval of time. They present a mathematical solution to this problem and hypothesize that the mechanisms that regulate stomata should approach, through natural selection, this theoretical optimum. There is a great deal of experimental work that lends support to this theory (for a review, see Farquhar and Sharkey, 1982).

The model presented here developed from attempts to apply the Cowan-Farquhar theory at the level of the leaf in order to understand the regulatory responses of stomata to specific factors of the environment. The theory requires that we consider the responses of both photosynthesis and transpiration in such an analysis. Work by Wong et al. (1979) and Ball and Berry (1982) led to the concept that the responses of stomata to changes in the environment can be partitioned into components that are dependent on photosynthesis and others that are independent of photosynthesis. For example, a change in light (which primarily affects the kinetics of photosynthesis), with all other factors held constant, generally causes  $g_s$  to respond so as to maintain a constant proportionality of  $g_s$  to  $A_n$ . On the other hand, a change that affects only the kinetics of water vapor diffusion (e.g. a change in the diffusion gra-

dient,  $\mathcal{D}$ ) generally causes the  $g_s/A_n$  ratio to change. The analysis of Ball (1988), which provides the basis for the model used here, includes an implicit dependence of  $g_s$  on photosynthesis.  $A_n$  was measured together with other relevant environmental parameters and included as an independent variable in analysing the responses of  $g_s$ . Most of the remaining variation in  $g_s$  could be attributed to effects of the  $\text{CO}_2$  and  $\text{H}_2\text{O}$  concentration of the air in the local environment of the stomata. In the system of models described here, we make use of a photosynthesis model based on the work of Farquhar et al. (1980) to obtain  $A_n$  as a function of environmental variables.

A significant feature of the present stomatal model is that it is structured to consider the influence of the laminar boundary layer (which affects the local environment of the stomata) on the regulatory responses of the leaf system. The leaf surface is considered as the boundary separating the physiological processes that operate within the leaf from the transport processes that operate in the boundary layer (see Fig. 1), and the physiological model was parameterized with reference to environmental conditions specified at the leaf surface (see Ball, 1987). It is important to note that this does not necessarily imply that the regulatory mechanisms of the stomata 'sense' the concentrations at the leaf surface. Rather, the use of surface conditions is a logical requirement for separating the transport of sensible heat,  $\text{CO}_2$  and  $\text{H}_2\text{O}$  through a laminar boundary layer of variable conductance and the physiological processes of the leaf.

The physiological models (for  $A_n$  and  $g_s$ ) are combined with a model giving the environment at the leaf surface and the leaf energy balance as a function

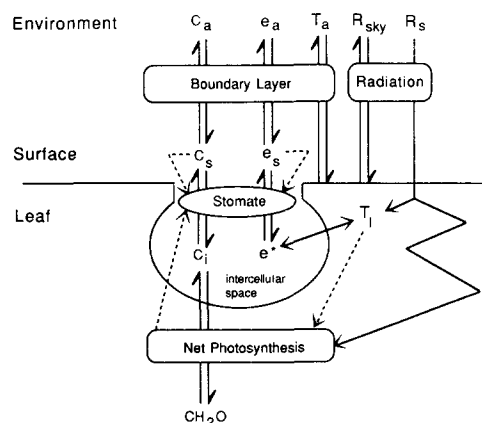


Fig. 1. A scheme showing the mass and energy fluxes along with the interactions between these considered in the complete model. Fluxes are shown as solid lines. Regulatory interactions are shown as dashed lines.  $c$ ,  $e$  and  $T$  stand for  $\text{CO}_2$ ,  $\text{H}_2\text{O}$  vapor concentrations and temperature, respectively. The subscripts  $a$ ,  $s$  and  $i$ , refer to properties in ambient air, at the leaf surface and of the leaf, respectively.  $R_{sky}$  specifies the long-wave length radiation input from the sky and  $R_{solar}$  represents solar radiation.

of  $g_s$ . These interacting models are solved by a numerical method, and we present simulations of the responses of  $g_s$  and latent heat flux ( $\lambda E$ ) of a soybean leaf in free air with the boundary layer treated as an independent variable.

The simulations illustrate the possible significance of the boundary layer in mediating feedback loops which affect the regulation of stomatal conductance and  $\lambda E$ . The simulations also examine the significance of changing the response properties of the photosynthetic component of the model by changing leaf protein content or the  $\text{CO}_2$  concentration of the atmosphere.

#### MODEL DESCRIPTION

Below we describe the models used and briefly discuss their parameterization. A general scheme showing the structures, processes, fluxes and regulatory interaction considered here is presented in Fig. 1. An Appendix to this article provides definition of terms, parameter values and equations used in our simulations of this system.

##### *Stomatal sub-model*

Ball (1988) and Ball and Berry (1991) proposed an equation to describe the response of  $g_s$  to the rate of net  $\text{CO}_2$  uptake ( $A_n$ ) and the relative humidity ( $h_s$ ) and  $\text{CO}_2$  mole fraction ( $c_s$ ) of the air at the leaf surface

$$g_s = m \frac{A_n h_s}{c_s} + b \quad (1)$$

The terms  $m$  and  $b$  are, respectively, the slope and intercept obtained by linear regression analysis of data from gas exchange studies with single leaves in a ventilated, temperature-controlled cuvette. Surface concentrations are functions of ambient concentrations, leaf boundary-layer conductance ( $g_b$ ) and  $g_s$ . These are routinely evaluated in gas exchange studies, as described by Ball (1987). In this paper, we will follow the convention of units introduced by Cowan (1977) and in common use in the physiological literature. Conductances are specified in units of  $\text{mol m}^{-2} \text{s}^{-1}$  and the diffusion gradients are given in mole fraction (i.e.  $\text{mol mol}^{-1}$ ). For convenience, a conductance of  $1 \text{ cm s}^{-1}$  is approximately  $0.4 \text{ mol m}^{-2} \text{s}^{-1}$  at  $25^\circ\text{C}$  and an atmospheric pressure  $P = 101.3 \text{ kPa}$ . The flux of  $\text{CO}_2$  uptake,  $A_n$ , is in units of  $\mu\text{mol m}^{-2} \text{s}^{-1}$ ,  $c_s$  is in  $\mu\text{mol mol}^{-1}$ ,  $h_s$  is a unitless ratio,  $m$  is unitless and  $b$  is in conductance units.

Equation (1) was developed from a series of gas exchange studies described in detail by Ball and Berry (1991). Briefly, these studies were structured to examine the dependence of  $g_s$  on each of the independent variables  $A_n$ ,  $c_s$  and  $h_s$ . Since it was not practical to always arrange conditions to keep  $A_n$  constant while other variables were changed, a sequential analysis protocol

was adopted. First, building on the work of Wong and others (Wong et al., 1978, 1979, 1985a,b,c), it was established that  $g_s$  varies in proportion to  $A_n$  when  $c_s$  and  $h_s$  were held constant. This permitted  $A_n$  to be brought to the left side of the equation (e.g. eqn. (1)), so that the effect of  $h_s$  and  $c_s$  could be examined against  $g_s/A_n$  as the dependent variable. Second, it was observed that the value of the proportionality constant varied according to consistent patterns when either  $c_s$  or  $h_s$  was changed: (a)  $g_s/A_n$  varied in proportion to  $1/c_s$  provided  $h_s$  was held constant; (b)  $g_s/A_n$  varied in direct proportion to  $h_s$  with  $c_s$  constant. Finally, it was established that the variables appeared to interact multiplicatively over physiologically interesting ranges of values.

The use of  $h_s$ , as opposed to the actual vapor pressure gradient ( $\mathcal{D}_s$ ), was decided from a series of experiments which examined the response to  $\mathcal{D}_s$  at different temperatures. Studies of Ball and Berry (1982, 1991) indicated that the sensitivity of  $g_s/A_n$  to  $\mathcal{D}_s$  decreased with increasing temperature (see Fig. 5), and they found that this could be approximated as  $g_s/A_n \propto (1 - \mathcal{D}_s/e^*)$  where  $e^*$  is the saturation vapor pressure of water at  $T_l$ . The term  $(1 - \mathcal{D}_s/e^*)$  can be simplified to relative humidity,  $h_s$ . At the present time, we view the use of  $h_s$  in eqn. (1) as a mathematical 'convenience'. There is no direct evidence for a humidity sensor. In fact, recent studies by Mott and Parkhurst (1991) indicate that the response of  $g_s$  at any constant temperature is probably controlled by the rate of transpiration, but they also observed that the sensitivity of  $g_s$  changed with temperature approximately as described by eqn. (1).

Norman and Polley (1989) and Ball and Berry (1991) reported regression parameters for several  $C_3$  and  $C_4$  species. The parameters of eqn. (1), i.e. the slope factor  $m$  and intercept  $b$ , may be determined from a linear regression of  $g_s$  as a function of the product  $A_n h_s / c_s$ , where these terms are obtained from measurements of leaf gas exchange conducted with intact attached leaves under controlled conditions. To be assured that the regression is appropriately weighted, the data should include a range of  $CO_2$  concentrations, light intensities, temperatures and atmospheric humidities that span the range of interest. Ball and Berry (1991) caution that eqn. (1) should not be used when  $A_n \rightarrow 0$  (at  $Q_p < 50 \mu\text{mol m}^{-2} \text{s}^{-1}$  or  $c_s < 100 \mu\text{mol mol}^{-1}$ ), and data in these ranges should not be included in the linear regression analysis. Leuning (1990) found that using  $c_s - \Gamma$ , where  $\Gamma$  is the  $CO_2$  compensation concentration, gave a better fit at low values of  $c_s$ .

### *Photosynthesis sub-model*

Prediction of  $g_s$  according to eqn. (1) requires that  $A_n$  be known.  $A_n$  is a useful quantity to know since it is directly related to the primary productivity of leaves and canopies, but its relevance in this discussion is limited to the dependence of  $g_s$  on  $A_n$ .

The  $\text{CO}_2$ , light and temperature dependence of  $A_n$  (following Farquhar et al. (1980) and Kirchbaum and Farquhar (1984)) is estimated as

$$A_n \approx \min \left\{ \begin{array}{l} J_E, f(Q_p, a, p_i, T_l) \\ J_C, f(V_m, p_i, T_l) \\ J_S, f(T_l, V_m) \end{array} \right\} - R_d, f(T_l, V_m) \quad (2)$$

where the terms  $J_E$ ,  $J_C$  and  $J_S$  are separate rate expressions for photosynthetic  $\text{CO}_2$  uptake in terms of a different rate-limiting step in the overall process of photosynthesis ( $J_E$ , light;  $J_C$ , ribulose biphosphate (RuBP) carboxylase (or Rubisco);  $J_S$ , sucrose synthesis), and  $R_d$  is the rate of 'day' respiration. The actual rate approaches the minimum (or most limiting) rate. In practice, the first term on the right in eqn. (2) is transformed to a continuous function with a smooth transition from one limiting factor to another using nested quadratic equations. These and the expressions for  $J_E$ ,  $J_C$  and  $J_S$  are given in the Appendix. The latter are functions of kinetic constants, leaf properties (i.e. the catalytic capacity,  $V_m$ , of the  $\text{CO}_2$ -fixing enzyme, Rubisco, and the absorptance to  $Q_p$ ,  $a$ ) and environmental parameters (i.e. the leaf temperature,  $T_l$ , the incident flux of photosynthetically active photons,  $Q_p$ , and the partial pressure of  $\text{CO}_2$  in the intercellular air spaces of the leaf,  $p_i$ ). Note that  $p_i = c_i \times P$ , where  $c_i$  is in mole fraction and  $P$  is the atmospheric pressure.

A complete listing of the kinetic parameters, their temperature dependencies and the equations used in the model is presented in the Appendix. The parameters that may vary from leaf to leaf are the absorptance to  $Q_p$ ,  $a$  (generally,  $a \sim 0.9$ ), and the maximum capacity of the carboxylase reaction ( $V_m$ ).

Selection of the appropriate value to use for the  $V_m$  of Rubisco is the major problem in parameterizing the photosynthesis model. For the simulations reported here, we estimated this by an *in vivo* assay. Gas exchange studies are used to measure the slope of the response of  $A_n$  to  $p_i$  ( $dA_n/dp_i$ ) at light saturation, at a known leaf temperature (about  $25^\circ\text{C}$ ) and at  $p_i = \Gamma_*$ .  $V_m$  may then be calculated according to eqn. (A6), as described in the Appendix. It is also possible in some cases to measure  $V_m$  directly by extracting leaf proteins and assaying the enzyme activity (see Woodrow and Berry, 1988). With appropriate calibration experiments, it is possible to estimate  $V_m$  from measurements of total leaf nitrogen since Rubisco is a significant and fairly constant fraction of total leaf protein (see Evans and Seemann, 1989).

### *Combining the sub-models*

These two sub-models may be viewed as being interdependent at the leaf level. The photosynthesis model requires  $p_i$  as an input variable, and this is a function of  $c_s$  and  $g_s$ . The latter can be obtained from the stomatal model, but this requires a value of  $A_n$ . A simultaneous solution can be obtained by numerical methods.

### Tests of the models

Figure 2 shows a plot of  $g_s$  predicted from eqn. (1) compared with measured values of  $g_s$  obtained in 169 separate gas exchange measurements with several different leaves (see Ball (1987) and Ball and Berry (1991) for experimental details). These measurements were conducted under a wide variety of conditions ( $T_l=20-35^\circ\text{C}$ ,  $Q_p=100-2000\ \mu\text{mol m}^{-2}\text{s}^{-1}$ ,  $h_s=0.45-0.90$ ,  $c_s=100-900\ \mu\text{mol mol}^{-1}$ ). The high correlation between predicted and measured  $g_s$  ( $r^2=0.92$ ) illustrates the predictive power of this sub-model. The values of the slope factor  $m$  and intercept  $b$  were determined from an analysis of an independent set of measurements (Ball and Berry, 1990).

Figure 3 shows a plot of predicted vs. measured values of  $A_n$  from gas exchange studies of leaves of *Glycine max*. Predicted  $A_n$  was calculated from the photosynthesis sub-model using measured values of  $p_i$ ,  $Q_p$ , and  $T_l$  obtained at same time as the measurement of  $A_n$ . The value for  $V_m$  was estimated as described in the Appendix. Three groups of leaves from different growth regimes and having different  $V_m$  values were included in this study. The set of measurements is identical to those included in Fig. 2. The analysis shows a very good correspondence ( $r^2=0.93$ ) between the predicted and the measured values of  $A_n$ .

Figure 4 shows a plot of the predicted value for  $g_s$  obtained by solving the combined sub-models compared with the actual measured values. The correlation ( $r^2=0.90$ ) for the combined models provides strong empirical support for this approach to modeling  $g_s$ . A similar but independent test of a com-

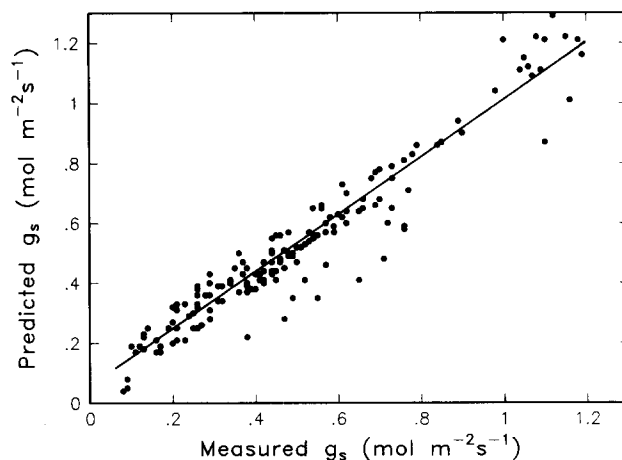


Fig. 2. Stomatal conductances measured in a leaf gas exchange cuvette, compared with those predicted by eqn. (1) using measured values of  $h_s$ ,  $c_s$  and  $A_n$ . The line drawn through the points is a linear regression of data ( $y=0.95x+0.05$ ,  $n=169$ ,  $r^2=0.92$ ).



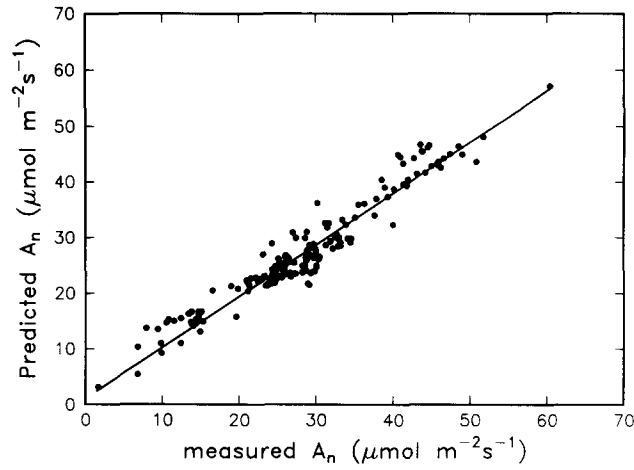


Fig. 3. Net photosynthetic rates measured in a leaf gas exchange cuvette compared with those predicted by the photosynthesis model. Measured values for  $c_i$ ,  $Q_p$ ,  $V_m$  and  $T_l$  were used as inputs of the model. The line represents a linear regression of the data ( $y=0.9x+1.89$ ,  $r^2=0.93$ ).

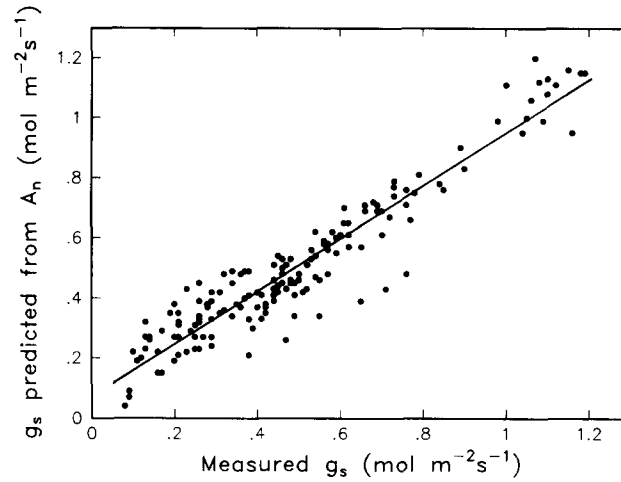


Fig. 4. Measured stomatal conductances compared with those predicted by the combined sub-models for net photosynthesis and stomatal conductance. Input parameters were  $h_s$ ,  $c_s$ ,  $Q_p$ ,  $V_m$  and  $T_l$ . The line is drawn from a linear regression of the data ( $y=0.87+0.07x$ ,  $r^2=0.90$ ).

bined photosynthesis model with the Ball and Berry stomatal model is presented by Leuning (1990).

#### *Comparison with other stomatal models*

The combination of the two sub-models (eqns. (1) and (2)) provides  $g_s$ ,

as a function of environmental variables, using prediction of  $A_n$  as an intermediate step. If  $A_n$  is not of interest (only  $g_s$ ), then we may regard the above model as essentially equivalent to other empirical models that give  $g_s$  directly as a function of environmental variables. Here, we will examine some of the similarities and differences that distinguish this model from other models.

First, as noted in the introduction, the Ball–Berry model is parameterized to be driven by the environmental conditions at the leaf surface, while other models (Jarvis, 1976; Avissar et al., 1985; Lindroth and Halldin, 1986) relate stomatal response to ambient conditions. It seems reasonable to assume that stomata respond to their local environment and to expect that this local environment should be influenced by the boundary layer of the leaf or canopy, and by the fluxes of heat,  $H_2O$  and  $CO_2$  at the leaf surface (Choudhury and Monteith, 1986). The Ball–Berry model is, thus, the only model structured to permit realistic treatment of these influences of  $g_b$  on the regulatory responses of stomata. There is no intrinsic reason, however, preventing other models from being parameterized to the surface environment.

Avissar and Mahrer (see Avissar et al., 1985) proposed a model that describes the attenuation of stomatal conductance from some maximum value by vapor pressure deficit of the air ( $\mathcal{D}_a$ ), ambient partial pressure of  $CO_2$ ,  $p_a$ ,  $T_l$  and total radiation ( $R_l$ ) using a threshold-type function. This form predicts that stomata tend to go from being fully open to completely closed over a narrow range of an environmental variable. We observe, however, that stomata generally respond continuously over a broader range of variable values. The Ball and Berry (1991; B–B), Jarvis (1976, J) and Lohammer (Lindroth and Halldin, 1986; L) models show more gradual responses to variables and only these latter models will be discussed in detail, keeping in mind that the Avissar–Mahrer model will agree with the others as  $g_s$  approaches its maximum or minimum values.

Both the L and J models describe  $g_s$  as a function of a maximum stomatal conductance ( $g_{max}$ ) by scaling this value with a series of factors (each between 0 and 1) which are functions of environmental variables taken separately (e.g. temperature,  $R_l$ ,  $\mathcal{D}_a$ ). The B–B model includes direct stomatal responses to  $CO_2$  and humidity, plus indirect responses to light,  $CO_2$  and temperature, through the effects of these variables on  $A_n$ . The model for  $A_n$  is structured such that strong interactions may occur between temperature, light and  $CO_2$ , while the L and J models differ in assuming that these factors act independently and are multiplicative. We will return to this point later. To begin our comparison, we note that the models are structurally similar in their treatment of the response of  $g_s$  to atmospheric humidity, and all three models can be written in the form

$$g_s = g'_s \times f(\mathcal{D}) + b \quad (3)$$

where  $g'_s$  is essentially the stomatal conductance under the conditions in ques-

tion in the absence of atmospheric drought,  $b$  is an intercept and  $f(\mathcal{D})$  is the  $\mathcal{D}$  response factor. For the B-B model, this factor is

$$f(\mathcal{D}) = h_s = 1 - \frac{\mathcal{D}_s}{e^*} \quad (4)$$

while for the J model

$$f(\mathcal{D}) = (1 - m' \mathcal{D}_a) \quad (5)$$

and the L model uses

$$f(\mathcal{D}) = \frac{1}{1 + m'' \mathcal{D}_a} \quad (6)$$

To appreciate the differences in these response functions, consider that a change in  $\mathcal{D}$  may be driven by changes in ambient vapor pressure ( $e_a$ ) or by a change in  $T_1$  which affects  $e^*$ , since  $\mathcal{D} = e^* - e_a$ . For the purpose of comparison, we will dispense with the distinction between  $\mathcal{D}_a$  and  $\mathcal{D}_s$  by assuming that  $g_b$  is large so that  $T_1 \rightarrow T_a$  and  $\mathcal{D}_s \rightarrow \mathcal{D}_a$ . Figure 5 compares the responses of  $g_s$  of a soybean leaf to a  $\mathcal{D}$  imposed by varying  $e_a$  at constant temperatures of 20 or 35°C, and by varying temperature (20–37°C) at constant  $e_a$ . At any constant temperature,  $g_s$  decreases linearly with increasing  $\mathcal{D}$ . This is consistent with the form of the J model (eqn. (5)), but as shown (Fig. 5), the slope of the response of  $g_s$  to an imposed  $\mathcal{D}$  (the value of  $m'$ ) is lower at 35°C than at 20°C. When  $\mathcal{D}$  is increased by increasing temperature at constant  $e_a$ ,  $g_s$  decreases hyperbolically (Fig. 5). This cannot be fit by a single value of  $m'$  (eqn. (5)). The empirical equation used in the L model (eqn. (6)) yields a

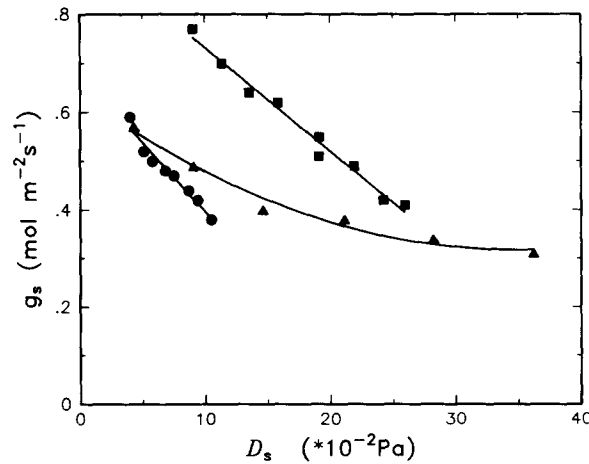


Fig. 5. The response of stomatal conductance to vapor pressure deficit imposed by changing the ambient or saturation vapor pressures; variable  $e_a$  at 20°C, (●); variable  $e_a$  at 35°C, (■); constant  $e_a$  at 20–37°C, (▲).

hyperbolic decrease in  $g_s$  with increasing  $\mathcal{D}$ . This approximates the response pattern observed with a temperature imposed  $\mathcal{D}$ , but it does not fit the constant temperature case. The curve shape of the L model may be related to the fact that it was developed from analysis of responses in a natural system (a forest canopy) where temperature was the major factor causing the  $\mathcal{D}_a$  to vary. The B-B model fits both types of curve shown in Fig. 5 ( $r^2=0.76$  for all data points). This is directly related to the presence of a temperature-dependent term ( $e^*$ ) in the  $\mathcal{D}$  response in the B-B model (eqn. (4)).

We now turn to the equations used to predict the value of  $g'_s$  of eqn. (3). This can include the response to a number of separate environmental variables. Using the terms of Jarvis (1976), this may be written as

$$g'_s = g_{\max} \times h(Q_p) \times i(T_1) \times j(c_a) \dots \quad (7)$$

where the terms  $h, i, j, \dots$  are factors between 0 and 1. The L model uses a similar approach, but does not include a response to  $c_a$  or  $T$ . From eqns. (1) and (3), we may obtain for the present model

$$g'_s = \frac{mA_n}{c_s} + \frac{b}{f(\mathcal{D})} \quad (8)$$

It should be noted that the appropriate value of  $A_n$  is that evaluated at the steady-state conditions ( $T_i$ ,  $Q_p$ ,  $p_i$ ) in question, and the value of  $A_n$  will vary with  $\mathcal{D}$  to the extent that the values of  $T_i$  or  $p_i$  change with  $g_s$ . Thus, we still require a solution to the combined photosynthesis and stomatal sub-models to evaluate  $g'_s$ .

The term  $g_{\max}$  of eqn. (7) is a hypothetical 'maximum stomatal conductance' that reflects intrinsic differences between leaves in stomatal density and size. In the present model, this parameter can be viewed as being directly related to the parameters that affect the maximum photosynthetic capacity of leaves, mostly the  $V_m$  activity of Rubisco, which is in turn related to the quantity of protein committed to photosynthetic functions. Thus, leaf-to-leaf variation in  $g_{\max}$  may be highly correlated with leaf protein.

The environmental response factors to  $Q_p$ ,  $T$  and  $c_a$  of eqn. (7) are implicitly included in the  $A_n$  term of eqn. (8) and, as noted earlier, the photosynthesis model predicts a high degree of interaction between these factors. For instance, the response of  $A_n$  to temperature is such that the shape of the response curve and the apparent temperature optimum changes with the level of  $Q_p$  and the value of  $p_i$ . On the other hand, the J model uses a response function that yields a more or less symmetrical response, falling as temperature is increased or decreased from the optimum temperature. This function can be made to approximate the shape and temperature optimum of the photosynthesis model under any particular values of  $p_i$  and  $Q_p$ , but the parameters of the curve would need to be changed if the environment changed. Similarly, the response of  $A_n$  to  $Q_p$  yields a curve that can be approximated by the

rectangular hyperbola used in the J and L models, but the parameters of this rectangular hyperbola would need to be changed if the temperature or  $p_i$  were changed. The interacting effects of temperature,  $Q_p$  and  $p_i$  on  $A_n$  are accurately depicted by photosynthesis model (see Fig. 3; Farquhar et al., 1980; Kirschbaum and Farquhar, 1984). Ball and Berry (1991) showed that responses of  $g_s$  are strongly correlated with  $A_n$ , and the data presented in Fig. 4 confirm that the approach taken here provides a good basis to predict  $g_s$  over a wide range of temperature,  $Q_p$  and  $p_i$ . It is also significant that the present model requires fewer adjustable parameters than the L or the J models.

#### *Energy and mass balance sub-models*

The model developed above requires knowledge of the value of several environmental parameters at the leaf surface. Under natural conditions, these are determined by the aerodynamic properties of the leaf, the wind speed, the fluxes of radiation and heat and gases exchanged between the leaf and its surroundings. Well-established equations for energy balance and mass transport can be used to describe this system as a function of  $g_s$ ,  $g_b$  and the ambient environment (see Appendix). When the three models are combined,  $\lambda E$  of a leaf or simple ('big-leaf') canopy can be calculated from environmental conditions ( $c_a$ ,  $e_a$ ,  $R_t$ ,  $T_a$  and  $g_b$ ) and the physiological characteristics of the surface ( $V_m$ ,  $m$  and  $a$ ).

There is a strong interaction between the sub-models of this system and a computer program was constructed to obtain a numerical solution. Using an initial guess for  $g_s$  (i.e.  $g_s = 1 \text{ mol m}^{-2} \text{ s}^{-1}$ ) and  $p_i$  (i.e.  $p_i = 15 \text{ Pa}$ ) the program obtains  $T_l$ ,  $A_n$ ,  $g_s$  and  $p_i$  by iteration, using the Newton–Raphson method, until  $p_i$  is stable. (A copy of the program can be obtained from the authors on request.)

#### RESULTS OF SIMULATIONS

The simulations we are about to describe depict the responses of a single-sided, horizontal leaf in free air. However, we will discuss these results in terms of canopy responses, making the assumption that canopy processes can be approximated as a 'big leaf'. We acknowledge that this is not strictly valid since we do not treat structural features or the heterogeneous environment of a real canopy. We are encouraged in making this simplification by Raupach and Finnigan (1989), who defend the position that, "single-layer models of evaporation from plant canopies are incorrect but useful, whereas multilayer models are correct but useless". We adopt their pragmatic approach here (but we acknowledge that more realistic models may yield different results).

Our goal in these simulations is to examine the interaction of the boundary layer with the regulatory properties of stomata in natural systems. Previous

treatments of the influence of the conductances of the boundary layer and the stomata on evapotranspiration of plant surfaces (Jarvis and McNaughton, 1986) treat  $g_s$  as a prescribed variable (set at a constant plausible value), while boundary-layer conductance is permitted to vary. In reality, the value of  $g_b$  has an influence over the environmental conditions at the leaf surface, and these in turn affect and are affected by  $g_s$ . Our model permits  $g_s$  to be treated as a dependent variable when  $g_b$  is varied.

We have chosen to present simulations of  $\lambda E$  and  $g_s$  over the course of a day rather than more abstract simulations where only boundary layer (for example) is varied. Our rationale is that this permits comparison to actual measurements of canopy or leaf fluxes which are generally reported as diurnal curves. Furthermore, many analyses of the dependence of canopy processes on environmental variables are extracted from analysis of such curves. Our simulations provide a basis to consider the significance of apparent responses of the system, given the fact that parameters such as  $R_s$ ,  $T_a$ ,  $\mathcal{D}_a$  and  $g_b$  may covary over the course of a typical day.

Figure 6 shows the daily cycle of  $R_s$ ,  $T_a$  and  $\mathcal{D}_a$  used in our simulations. This is an idealized day, designed to represent typical summer conditions in temperate North American agricultural regions. Peak values for  $R_s$ ,  $T_a$  and  $\mathcal{D}_a$  are  $900 \text{ W m}^{-2}$ ,  $32^\circ\text{C}$  and  $3.5 \text{ kPa}$ , respectively. During the day,  $R_s$  advances and declines symmetrically around noon; air temperature follows  $R_s$ , but lags by about 2 h;  $\mathcal{D}_a$  tracks the march of  $T_a$  since  $e_a$  remains essentially constant.

Simulated responses of  $\lambda E$ ,  $g_s$  and  $A_n$  to the daily march of environmental conditions (Fig. 6) at different imposed values of  $g_b$  ( $0.3$ ,  $2$  and  $8 \text{ mol m}^{-2} \text{ s}^{-1}$ ) are shown in Fig. 7. The maximum exchange of  $\lambda E$  (Fig. 7(a)) occurs at the intermediate value of boundary-layer conductance ( $2 \text{ mol m}^{-2} \text{ s}^{-1}$  – a

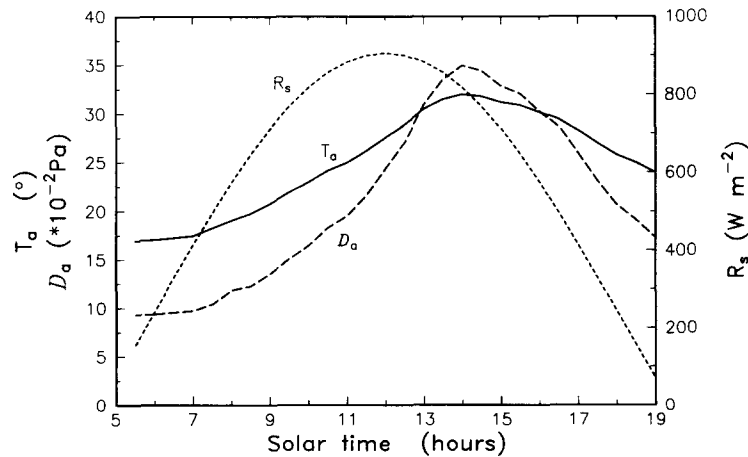


Fig. 6. The day courses of the environmental variables used in the simulations of latent heat flux and stomatal conductance.

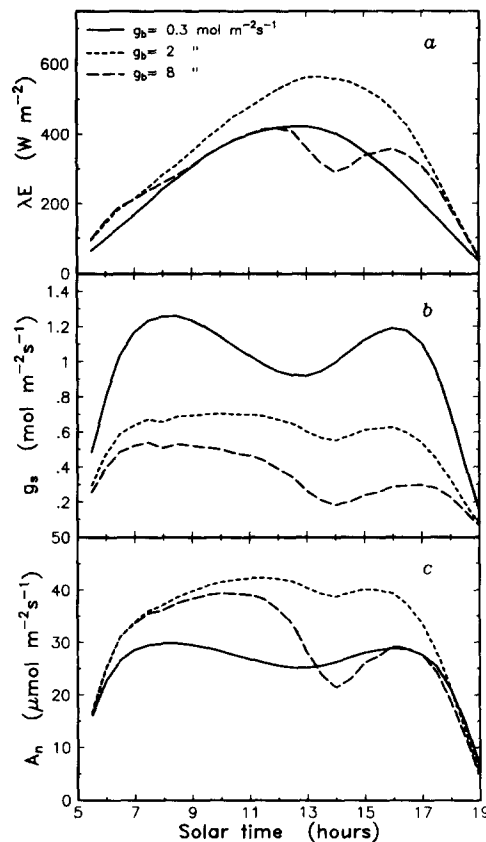


Fig. 7. Latent heat flux (a), stomatal conductance (b) and net photosynthesis (c) of a soybean canopy over the course of the day (Fig. 6) simulated at three values of boundary-layer conductances.

resistance of  $20 \text{ s m}^{-1}$ ). This value is approximately that expected for a well-developed crop canopy with moderate winds. Decreasing or increasing the boundary-layer conductance, as might occur with changes in wind speed, results in a substantial attenuation of  $\lambda E$ . The flux rises and falls with the changes in  $R_s$  and  $\mathcal{D}_a$ , except that a distinct midday depression is evident when the boundary layer is thin ( $g_b = 8 \text{ mol m}^{-2} \text{ s}^{-1}$ ). This is associated with midday stomatal closure (Fig. 7(b)) and a midday depression of photosynthesis (Fig. 7(c)). The depressions in  $g_s$  and  $A_n$  are not as strongly expressed in  $\lambda E$  at the lower values of  $g_b$ . This indicates that  $\lambda E$  is not likely to be a good indicator of physiological responses in closed canopies (Raupach and Finnigan, 1989).

Midday depressions of  $g_s$  affecting  $A_n$ , and occasionally  $\lambda E$ , have been observed in leaves (Beyschlag et al., 1986) and open natural canopies (Tan and Black, 1976; Cambell, 1989). The physiological models used in these simulations do not include any direct effect of the rate of transpiration or water

potential on photosynthesis. Such effects have often been invoked to explain the midday depression of  $\lambda E$  observed in the field. We cannot exclude the participation of such effects, but the present analysis shows that it is not necessary to postulate these responses to obtain midday stomatal closure.

The simulations show a strong interaction between the imposed value of  $g_b$  and the value of  $g_s$  selected by the regulatory properties of the model (leaf). In general,  $g_s$  increases as  $g_b$  is decreased (Fig. 7(b)). This response tends to resist changes in (stabilizes) the total conductance. The humidity response of the stomatal mechanism is primarily responsible for this effect. The humidity of the air at the leaf surface tends to increase as  $g_b$  is decreased because more of the transpired water vapor is retained by the boundary layer. Referring to eqn. (1), this should cause  $g_s$  to increase. This response has the characteristics of a 'feedforward' response (Farquhar, 1978) since  $\lambda E$  decreases substantially in going from 2 to 8 mol m<sup>-2</sup> s<sup>-1</sup>, while the evaporative demand imposed by the environment increases. It is interesting that the water-use efficiency ( $A_n/\lambda E$ ) is very similar at  $g_b=2$  and 8 mol m<sup>-2</sup> s<sup>-1</sup>, despite the higher evaporative demand at the higher value of  $g_b$  (compare Fig. 7(b) and 7(c)). Water-use efficiency is lowest at the low value of  $g_b$ , apparently because  $A_n$  is depressed.

The response of net photosynthesis is also of interest because it has a direct influence on stomatal conductance (see eqn. (1)). The strong depression of  $A_n$  during midday at  $g_b=8$  mol m<sup>-2</sup> s<sup>-1</sup> (Fig. 7(c)) is driven by stomatal closure and depletion of intercellular CO<sub>2</sub>. As noted, the closure of stomata may be initiated by the drying of the air at the leaf surface, but the decrease in  $A_n$  should have a positive feedback effect on  $g_s$ . The decrease in  $g_s$  caused by humidity leads to a decrease in  $A_n$ , that causes a further decrease in  $g_s$ .

The midday depressions of photosynthesis and  $g_s$  at low  $g_b$  (0.3 mol m<sup>-2</sup> s<sup>-1</sup>), are related by a different chain of cause and effect. The primary event leading to a decrease in  $g_s$  appears to be a decrease in  $A_n$  related to an increase in  $T_l$ , as the exchange of sensible and latent heat is restricted at low boundary-layer conductance. (Under these conditions,  $T_l$  is greater than the temperature optimum for photosynthesis.) According to eqn. (1), inhibition of  $A_n$  would lead directly to a depression of  $g_s$ . It should be noted however, that a decrease in  $g_s$  may also be expected to elicit a positive feedback on itself, mediated by the humidity response (humidity at the leaf surface would decrease if  $\lambda E$  decreases and if  $T_l$  increases).

Figure 8 shows a plot of the leaf to air temperature difference as a function of  $\mathcal{D}_a$  taken from a simulation of a daily curve at  $g_b=2$  mol m<sup>-2</sup> s<sup>-1</sup>. This type of plot has been widely used as an indicator of water stress in crop canopies (Idso et al., 1981) and, more recently, as a basis for estimating aerodynamic and crop resistances (O'Toole and Real, 1986). The approximately linear decrease in  $T_l - T_a$  occurs over the interval of 09:00–15:00 h and the slope is  $-1.9^\circ\text{C kPa}^{-1}$ . The pattern of response and the value of the slope is similar



to field observations made on several crop canopies (see O'Toole and Real, 1986). Analysis of these data using the approach of O'Toole and Real (1986) gives the mean values of  $g_b = 2.3$  and  $g_s = 0.6 \text{ mol m}^{-2} \text{ s}^{-1}$ , while the actual values were somewhat different (see Fig. 7(b)). Faver and O'Toole (1989) have also noted that estimates of  $g_b$  from aerodynamic considerations are lower than those derived from analysis of plots similar to Fig. 8. In our simulations,  $g_s$  is not constant (see Fig. 7(b)), as their analysis assumes. Idso (1987) has used the existence of a linear response of  $T_l - T_a$  to  $D_a$  (similar to Fig. 8) to argue that the conductance of crop canopies does not respond to humidity. The model used here includes a humidity response of  $g_s$ . Our simulations show that the linear response is not inconsistent with humidity control of  $g_s$ . tests of the accuracy of these simulations await more complete quantitative measurements of canopy fluxes and energy balance from systems that have been physiologically characterized.

In Fig. 7,  $g_b$  is treated as an independent variable that is held constant over the course of the day, when in reality  $g_b$  is dependent on other system variables and may change (over seconds or hours with changes in ventilation, and seasonally with changes in canopy structure). By plotting  $g_b$  and time as independent variables in three dimensions, we see a more complete picture of the dependence of  $\lambda E$  on  $g_b$  (Fig. 9). In these simulations, we choose a range of  $g_b$  ( $0.1\text{--}10 \text{ mol m}^{-2} \text{ s}^{-1}$ ) wider than is likely to occur in any natural canopy subject to high irradiance. The extremes may be approached by small leaves of open well-ventilated systems (e.g. a desert shrub) or by the large leaves of certain shade species (e.g. a banana tree) (see Grace, 1983), and

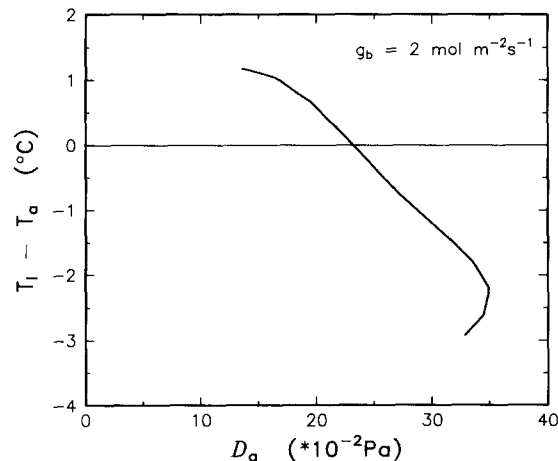
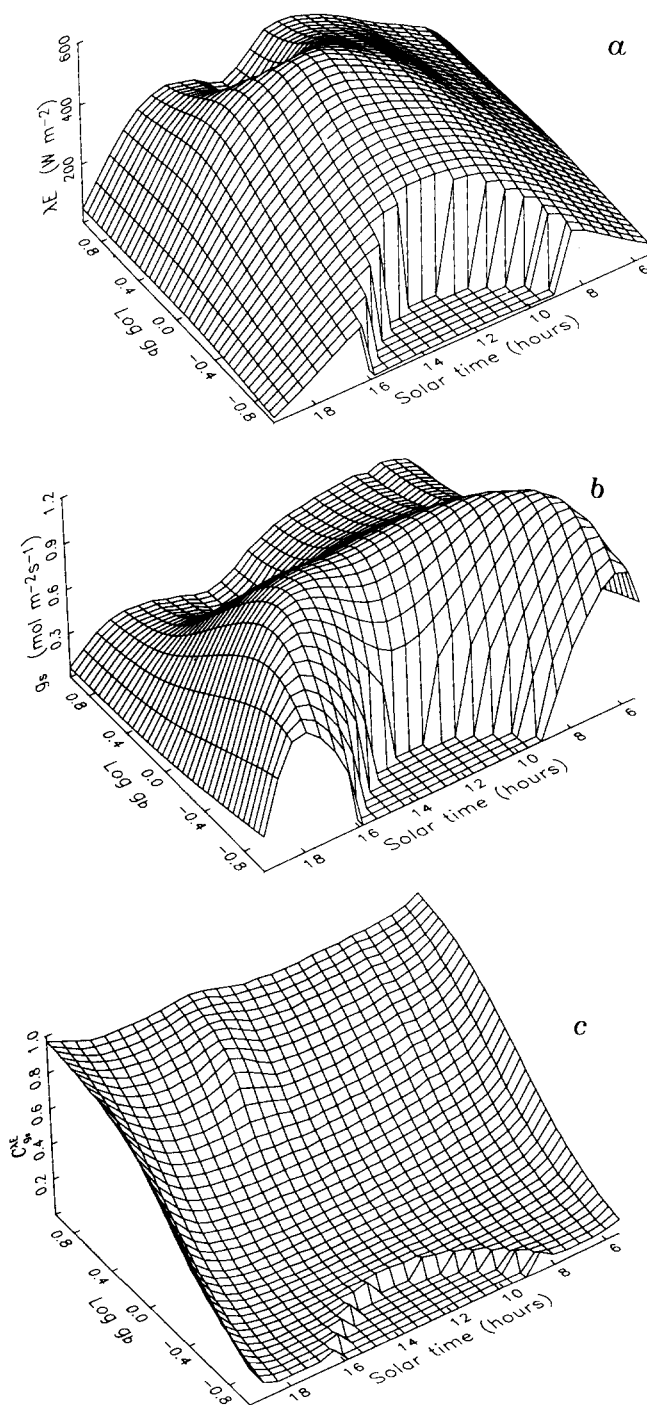


Fig. 8. The leaf temperature—air temperature difference plotted against the vapor pressure deficit of the air. Data taken from 09:00 to 15:00 h of a simulated day curve (Fig. 7) at  $g_b = 2 \text{ mol m}^{-2} \text{ s}^{-1}$ .



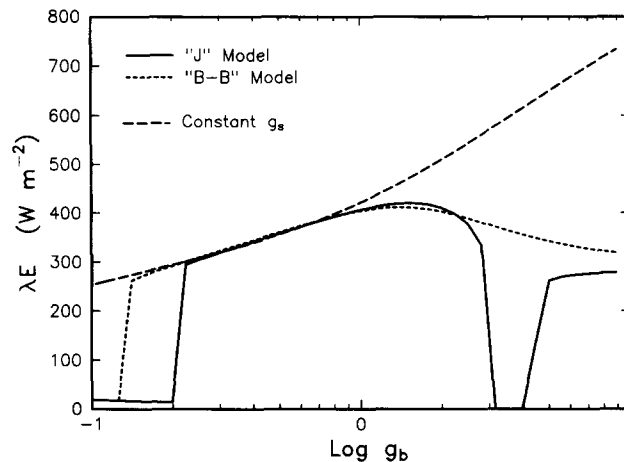


Fig. 10. Simulations of latent heat flux as a function of log boundary-layer conductance for different physiological responses. Environmental conditions are  $R_s = 800 \text{ W m}^{-2}$ ,  $e_a = 10.5 \text{ kPa}$  and  $T_a = 25^\circ \text{C}$ . Two forms of the coupled photosynthesis, stomatal conductance model are shown, one using the Ball–Berry humidity response (B–B) (eqn. (1)) and the other using the Jarvis type humidity response (J) using eqn. (5) with  $g'_s$  as given by eqn. (8). For comparison, a plot of latent heat flux at a constant stomatal conductance ( $1 \text{ mol m}^{-2} \text{ s}^{-1}$ ) is shown.

the responses at extreme values of  $g_b$  are useful for interpreting the feedbacks within the system.

Figure 10 shows a transect through the  $\lambda E$  surface (Fig. 9(a)) at about 12:00 h, and compares the result with simulations obtained using altered forms of the stomatal-photosynthesis model. To assist in interpreting the response of  $\lambda E$  to  $g_b$ , we present a three-dimensional plot of  $g_b$  (Fig. 9(b)) and a plot (Fig. 9(c)) showing the sensitivity of the modeled flux of  $\lambda E$  to  $g_s$  ( $C_{g_s}^{\lambda E} = (\partial \lambda E / \partial g_s) \times (g_s / \lambda E)$ , determined by finite difference). The latter parameter is a 'control coefficient' that indicates the extent to which the output of the system is controlled by  $g_s$  at the operating point. In general,  $0 < C < 1$ , and a value of  $C = 1$  indicates that the flux is proportional to the variable in question (i.e. it controls the flux) (see Woodrow, 1989; Woodrow et al., 1990).

#### Stomatal control of $\lambda E$

Jarvis and McNaughton (1986) discussed the control that stomata have on  $\lambda E$  in the context of the linearized energy balance model of Penman–Mon-

Fig. 9. (a) A three-dimensional plot of  $\lambda E$  as a function of the time of day and the log  $g_b$ . (b) The corresponding plot of stomatal conductance. (c) A plot of the sensitivity of latent heat flux to stomatal conductance ( $C_{g_s}^{\lambda E}$ ). Latent heat flux tends to be independent of stomatal conductance when  $C_{g_s}^{\lambda E} < 0.30$ , whereas higher values indicate regions of strong stomatal control of latent heat flux. The values of  $C_{g_s}^{\lambda E}$  are truncated to 0 at low values of  $g_b$  where  $T_l$  reaches lethal values.

teith in which  $g_s$  and  $g_b$  are independent variables, and they define a parameter  $\Omega$  ( $\Omega = 1 - (d\lambda E/dg_s) \times (g_s/\lambda E)$ ) that indicates the extent to which  $\lambda E$  approaches the limit of 'equilibrium evaporation'. It should be noted that  $\Omega$  is similar to  $1 - C_{g_s}^{\lambda E}$ , as used here, with the exception that  $g_s$  is a dependent variable in our analysis. The consequence of permitting  $g_s$  to vary as predicted by the complete model is that ' $\Omega$ ' (which is primarily a function of the ratio  $g_s/g_b$ ) is also dependent on the regulatory responses that affect the value of  $g_s$ . Referring to Fig. 9(c), we can see that ' $\Omega$ ' (1 – the plotted value) varies over the course of a day.  $C_{g_s}^{\lambda E}$  tends to be larger in the morning and evening (stomata are more in control) than at midday, and  $C_{g_s}^{\lambda E}$  increases under conditions where midday stomatal closure occurs.  $C_{g_s}^{\lambda E}$  becomes larger (stomata exert more control) as the boundary-layer conductance increases, and it increases again as  $g_b \rightarrow 0.1 \text{ mol m}^{-2} \text{ s}^{-1}$  where  $g_s \rightarrow 0$  (when  $g_s = 0$ ,  $C_{g_s}^{\lambda E}$  is undefined and truncated to 0 in Fig. 9(c)). At  $g_b < 1 \text{ mol m}^{-2} \text{ s}^{-1}$ ,  $C_{g_s}^{\lambda E}$  is generally small, indicating (as proposed by Jarvis and McNaughton (1986)) that stomata exert little control over  $\lambda E$  when  $g_b$  is low. Nevertheless, very substantial changes in  $g_s$  occur over this range of  $g_b$  (see Fig. 9(b)). It may be unwise to dismiss these stomatal responses as having no significance. We note that the opening of stomata when  $g_b$  is low tends to decrease  $T_l$  and increase  $p_i$ , both of which may have substantial effects on photosynthesis. This is an area which needs further attention.

#### *Discussion of midday responses*

For the purpose of discussion, and referring to Figs. (7), (9) and (10), we can distinguish three different types of midday response.

(1) In the intermediate range of  $g_b$  ( $0.2\text{--}1 \text{ mol m}^{-2} \text{ s}^{-1}$ ),  $\lambda E$  appears to track the input of  $R_s$  and increases as  $g_b$  increases (Figs. 9(a) and (10)). This is opposite to the response of  $g_s$  which tends to decrease while  $\lambda E$  increases (compare Fig. 9(a) and 9(b)). This corresponds to a region where  $C_{g_s}^{\lambda E} < 0.4$  (Fig. 9(c)), indicating that the flux is relatively insensitive to  $g_s$ . Figure 10 shows that  $\lambda E$  over this range of  $g_b$  is insensitive to the form of the sub-model used to specify  $g_s$ . For example, assuming that  $g_s = 1 \text{ mol m}^{-2} \text{ s}^{-1}$  yields nearly identical  $\lambda E$  over this range of  $g_b$ . In this region, the  $\lambda E$  approaches that of a wet surface. Some feedback regulation of  $g_s$  occurs as a result of temperature and humidity interactions (Fig. 9(b)), but the physiological system does not become unstable, and  $T_l$  and  $g_s$  co-vary so that  $\lambda E$  tracks  $R_s$ .

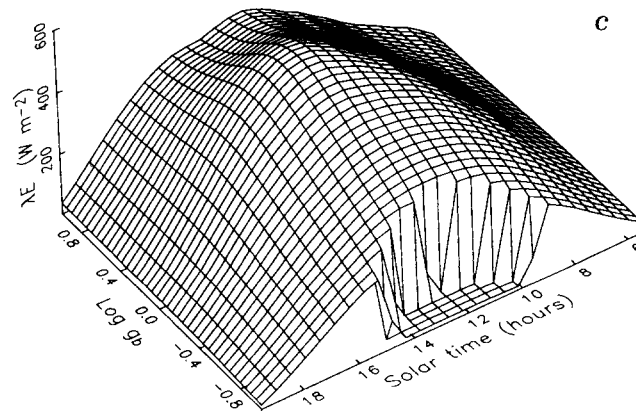
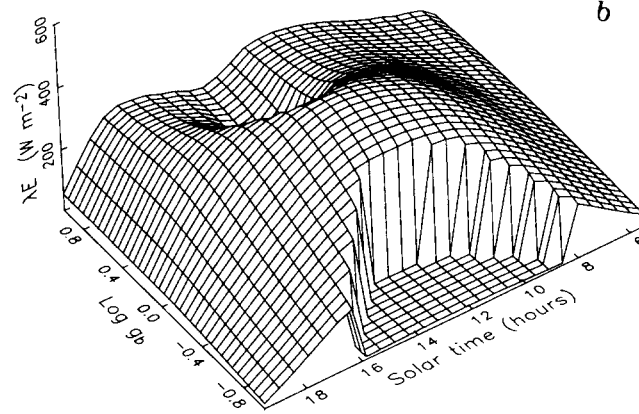
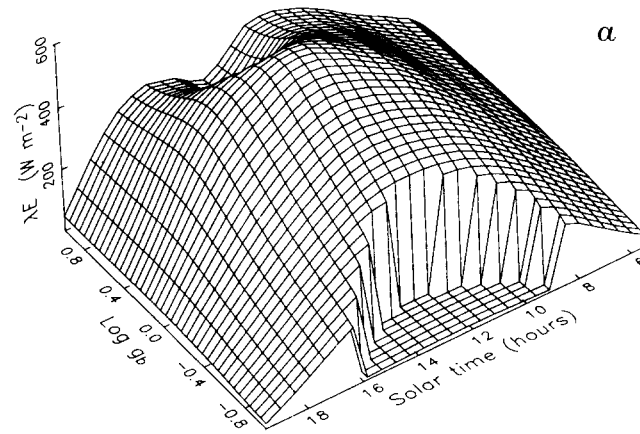
(2) At low  $g_b$  ( $< 0.2 \text{ mol m}^{-2} \text{ s}^{-1}$ ), there is precipitous closure of stomata during midday (Figs. 9(b) and 10)). Examination of model output (data not shown) reveals that leaf temperatures rise beyond levels known to cause permanent damage to the photosynthetic reactions of leaves. The total collapse of the physiological component in this region of  $g_b$  is apparently driven by the tendency for heat to be trapped within the boundary layer at low values

of  $g_b$ , as sensible and latent heat fluxes are restricted. Consequently, leaf temperature increases as  $g_b$  decreases and when  $T_l$  exceeds the temperature optimum for photosynthesis,  $A_n$  and  $g_s$  begin to fall. This response is accelerated by positive feedback mediated by a decrease in  $h_s$  and increase in  $T_l$  as  $g_s$  is decreased. Eventually, the physiological system becomes unstable with  $T_l$  approaching  $70^\circ\text{C}$ . Similar feedbacks affecting  $T_l$  have been observed in the laboratory and in the field as  $T_l$  approaches the lethal limit (Gamon and Pearcy, 1989). In the real world, there would be strong selective pressure against leaf properties that would lead to such high values of  $T_l$ . In Fig. 10, we show that the abrupt stomatal closure occurs at higher  $g_b$  (e.g. stronger feedbacks) if we introduce the humidity response of the J model (eqn. (5)) which does not include changes in the response to  $\mathcal{Q}_s$  with changes in  $T_l$ .

(3) At values of  $g_b$  exceeding about  $2 \text{ mol m}^{-2} \text{ s}^{-1}$ , a midday depression in  $\lambda E$  begins to appear (Figs. 9(a) and (10)). This is associated with a decline in  $g_s$  (Fig. 9(b)) and an increase in the sensitivity coefficient for  $\lambda E$  as a function of  $g_s$  (Fig. 9(c)). This depression in  $\lambda E$  flux is not seen when the humidity response components are eliminated from the model, but it becomes more pronounced when the  $\mathcal{Q}_s$  response (J model) is substituted for the  $h_s$  (Fig. 10). Thus, this response is related to the regulation of  $h_s$  by atmospheric humidity.  $\mathcal{Q}_a$  and  $T_l$  peak near 14:00 h (Fig. 6) and  $h_s$  should tend to reach a minimum at this time. The value of  $h_s$  should decrease as  $g_b$  increases, eventually approaching the humidity of the ambient air. The model includes an additional feedback arising from the decrease in  $A_n$  (see Fig. 7(c)) and  $p_i$  falls. The existence of positive feedback loops tends to make the system unstable with respect to changes in  $g_s$  (i.e. a perturbation that would cause  $g_s$  to increase or decrease can elicit other changes in the system which tend to amplify the response). In this regard, it is interesting to note that a characteristic of the B-B model tends to add stability to the system. This can be seen by comparing the responses at extreme values of  $g_b$  with either the J model or the B-B model (see Fig. 10). When compared with a simple response to  $\mathcal{Q}$ , the temperature-compensated response of the B-B model (see Fig. 5) tends to decrease positive feedback (involving humidity), leading to stomatal closure, and tends to increase that leading to opening of stomata.

#### *Effects of physiological capacity and $\text{CO}_2$ concentration*

The capacity of physiological components of this system and the value of  $c_a$  also influence the response properties, as illustrated in Fig. 11. For example, a simulation where  $V_m$  was reduced by 33% is shown in Fig. 11(b). The control, Fig. 11(a), is identical to Fig. 9(a).  $\lambda E$  is not affected when  $\mathcal{Q}_p$  is low in the morning and evening, but is lower during midday. The depressions in  $\lambda E$  at high and low  $g_b$ , seen in Figs. 7 and 9(a), are expanded. With further reduction in  $V_m$  (not shown), the two depressions merge so that stomata al-



ways close in high  $R_s$ , regardless of the value of  $g_b$ . The effect can be reversed by lowering  $R_s$  or increasing the value of  $m$ . Plants with low photosynthetic capacity should, therefore, not occur in high-radiation environments or, if they do, they should not have large leaves. Forest understory leaves are typically large and have low photosynthetic capacities, but of course they operate in environments characterized by low radiation and low  $\mathcal{D}$ .  $V_m$  may change according to environmental and physiological conditions on time scales from weeks to years in response to average light intensity (Wong et al., 1985a), soil nitrogen (Wong et al., 1985a) and water stress (Wong et al., 1985c). The simulations shown here indicate that changes in the physiological properties of the leaves making up a canopy may affect the exchange of  $\lambda E$ .

The  $\text{CO}_2$  concentration of the atmosphere has risen approximately by 70 ppm this century and the  $\text{CO}_2$  concentration may double over present levels in the next century. It is of some interest, therefore, to examine how higher  $\text{CO}_2$  concentrations might affect the energy exchange of plant canopies. Figure 11(c) shows a simulation with  $c_a$  twice the present level (all else identical to Fig. 11(a)). The simulation shows that increased  $\text{CO}_2$  causes a small decrease in  $\lambda E$  at intermediate values of  $g_b$  and it tends to result in increased  $\lambda E$  at the extremes of  $g_b$  (where midday depressions occur in normal  $c_a$ ). This effect is not intuitively obvious. Consider eqn. (1), increasing  $c_s$  should cause  $g_s$  to decrease, but in the complete model  $g_s$  is in fact increased by increased  $\text{CO}_2$ , at least over some ranges of  $g_b$ . This occurs because increasing  $\text{CO}_2$  tends to eliminate the positive feedback loop in  $g_s$  that is mediated by effects on  $p_i$  and  $A_n$ . At higher  $\text{CO}_2$ ,  $A_n$  is higher and is less sensitive to changes in  $p_i$ . It will be interesting to see if this occurs in real systems.

## CONCLUSIONS

The system of models developed here permits simulation of the transpiration and energy balance of a simple canopy using physiologically driven sub-models for photosynthesis and regulation of stomatal conductance. The predictive capacity of the physiological sub-models was examined in extensive gas exchange studies with attached intact leaves of soybean, and the tests show the model can be applied to a wide range of environmental conditions. The physiological models were included in a leaf energy balance and mass transport model structured to permit stomatal responses to be driven by the conditions in the local region of the stomata (i.e. at the leaf surface) through a boundary layer of air adjacent to the leaf. Simulations indicate that the con-

Fig. 11. Three-dimensional plots showing the effect of changing the  $V_m$  Rubisco and the concentration of  $\text{CO}_2$  on the flux of latent heat flux as a function of time of day and log boundary-layer conductance. (a) Control, identical to Fig. 9(a). (b) An identical simulation with  $V_m = 0.67$  times the control value. (c) An identical simulation with  $c_a$  twice the control value.

ductance of a leaf boundary layer can have significant influences on the canopy response properties. For example, simulation of a day curve (using the same ambient conditions and physiological properties) assuming different values of  $g_b$ , results in very different patterns. With a low  $g_b$ ,  $\lambda E$  is nearly proportional to  $R_s$  over the entire day, while with  $g_b = 8 \text{ mol m}^{-2} \text{ s}^{-1}$ , a strong midday depression of  $\lambda E$  is observed. Further examination of the response to  $g_b$  shows that midday stomatal closure can be driven by excessively high leaf temperature (at low values of  $g_b$ ) and by drying of the air at the leaf surface (at high values of  $g_b$ ). These responses interact with the photosynthetic capacity of the leaf (which affects  $g_{\text{max}}$ ), with the radiation load on the canopy, and with the  $\text{CO}_2$  concentration of the air. Similar response patterns are known to occur with natural vegetation of different types. The models presented here provide a plausible mechanism for these responses. Experimental studies at the canopy level are required to test the mechanisms proposed here.

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## APPENDIX

*Photosynthesis model*

Leaf photosynthesis is described here as the minimum of three potential capacities or

$$A \approx \min \begin{cases} J_E \\ J_C \\ J_S \end{cases} \quad (A1)$$

where  $A$  is the rate of gross  $\text{CO}_2$  uptake, and  $J_E$  and  $J_C$  are defined according to Farquhar et al. (1980).

$J_E$  describes the response of photosynthesis to  $Q_p$  as

$$J_E = a \times \alpha \times Q_p \frac{p_i - \Gamma_*}{p_i + 2\Gamma_*} \quad (A2)$$

where  $a$  is leaf absorptance to photosynthetically active radiation,  $\alpha$  is the intrinsic quantum efficiency for  $\text{CO}_2$  uptake, and  $\Gamma_*$  is defined by the equation

$$\Gamma_* = \frac{[\text{O}_2]}{2\tau} \quad (A3)$$

where  $\tau$  is a ratio of kinetic parameters describing the partitioning of RuBP to the carboxylase or oxygenase reactions of Rubisco.  $\tau$  can be determined experimentally from gas exchange experiments on intact leaves (see Brooks and Farquhar, 1985) or from enzymatic analysis in vitro (see Jordan and Ogren, 1981).  $[\text{O}_2]$  is assumed constant ( $209 \text{ mmol mol}^{-1}$ ),  $p_i$  is given by  $p_i = P \times c_i$ , and

$$c_i = c_a - A_n \frac{1.6g_s + 1.4g_b}{2.24g_s g_b} \quad (A4)$$

Note that  $c_a$ ,  $c_i$  and diffusion gradients are in mole fraction, while the kinetic expressions for biochemical reactions require partial pressure, and  $P$  is the atmospheric pressure.

$J_C$  is the Rubisco-limited rate and is defined as

$$J_C = \frac{V_m(p_i - \Gamma_*)}{p_i + K_c(1 + [\text{O}_2]/K_o)} \quad (A5)$$

where  $K_c$  and  $K_o$  are the Michaelis constant for  $\text{CO}_2$  and the competitive inhibition constant for  $\text{O}_2$  with respect to  $\text{CO}_2$  in the Rubisco reaction, and  $V_m$  is the maximum catalytic capacity of Rubisco per unit leaf area ( $\mu\text{mol m}^{-2} \text{ s}^{-1}$ ). For the calculations presented in Figs. 3 and 4,  $V_m$  was determined from measurements of the slope of the response of  $A_n$  to  $p_i$  at  $p_i = \Gamma_*$ , according to

$$V_m = \frac{dA_n}{dp_i} \left[ \Gamma_* + K_c \left( 1 + \frac{[O_2]}{K_o} \right) \right] \quad (A6)$$

(see Collatz et al., 1990).

$J_S$  is the capacity for the export or utilization of the products of photosynthesis (most likely sucrose synthesis, see Woodrow and Berry, 1988), and this is approximately the maximum value of  $A_n$  at saturating  $Q_p$  and  $CO_2$ . Here, we use

$$J_S = V_m/2 \quad (A7)$$

Equation (A1) is equivalent in form to that proposed by Kirchbaum and Farquhar (1984), but our definitions of  $J_C$ ,  $J_E$  and  $J_S$  differ somewhat from theirs.

To introduce a more realistic, gradual transition from one limitation to another, and to allow for some co-limitation between  $J_E$ ,  $J_C$  and  $J_S$ , we solve the following two quadratics for their smaller roots.

$$\theta J_P^2 - J_P(J_E + J_C) + J_E J_C = 0 \quad (A8)$$

and

$$\beta A^2 - A(J_P + J_S) + J_P J_S = 0 \quad (A9)$$

where  $A$  is the gross rate of  $CO_2$  uptake, and  $J_P$  is an intermediate variable that gives the minimum of  $J_E$  and  $J_C$ .  $\theta$  and  $\beta$  are empirical constants describing the transition between limitations, and are typically close to one (here we use 0.98 and 0.95, respectively).

Respiratory  $CO_2$  production ( $R_d$ ) was scaled to the  $V_m$  as

$$R_d = 0.015 V_m \quad (A10)$$

(see Farquhar et al., 1980) and  $A_n$  is then defined as

$$A_n = A - R_d \quad (A11)$$

Some of the kinetic parameters of the model,  $K_c$ ,  $\tau$ ,  $K_o$ ,  $V_m$  (and the fluxes  $R_d$  and  $J_S$ ), change with temperature. We used the  $Q_{10}$  function

$$k = k_{25} Q_{10}^{(T-25)/10} \quad (A12)$$

where  $k_{25}$  is the parameter value at  $25^\circ C$  and  $Q_{10}$  is the relative change in the parameter for a  $10^\circ C$  change in temperature. Values for the parameters and respective  $Q_{10}$  are given in Table A1 (see also Woodrow and Berry, 1988).

The combined temperature response of  $A_n$  described above does not account for the well-known thermal inhibition at temperatures exceeding  $35^\circ C$  (Berry and Raison, 1981). In order to simulate realistic rates at high temperature, we introduced to the model a gradual temperature inhibition of the  $V_m$  described by

$$V_m = V_m^o \left\{ 1 + \exp \left[ \frac{-a + b(T_1 + 273)}{R(T_1 + 273)} \right] \right\}^{-1} \quad (\text{A13})$$

where the superscript o indicates an intermediate value, corrected according to eqn. (A12) to  $T_1$  (see Weis and Berry, 1988).  $R$  is the ideal gas constant, and  $a$  and  $b$  are constants.

$R_d$  is also inhibited at high temperature and for this response we use the following form

$$R_d = R_d^o \{ 1 + \exp [ 1.3(T_1 - 55) ] \}^{-1}$$

which predicts an abrupt collapse in  $R_d$  as  $T_1$  approaches  $55^\circ\text{C}$  (see Björkman et al., 1980).

TABLE A1

Values of kinetic parameters and constants used in the models of stomatal conductance, photosynthesis and energy balance<sup>1</sup>

| Symbol           | Value  | Units                                | $Q_{10}$ | Description   |
|------------------|--------|--------------------------------------|----------|---|
| $a$              | 220    | $\text{KJ mol}^{-1}$                 |          | (Equation (A13))  |
| $a$              | 0.86   |                                      |          | Leaf absorptance to $Q_p$ (eqn. A2))                    |
| $b$              | 703    | $\text{J/mol } ^\circ\text{K}$       |          | (Equation (A13))  |
| $b$              | 0.01   | $\text{mol m}^{-2} \text{s}^{-1}$    |          | Intercept, B-B model (eqn. (1))                         |
| $c_a$            | 340    | $\mu\text{mol mol}^{-1}$             |          | Ambient mole fraction $\text{CO}_2$                     |
| $J_s$            | 100    | $\mu\text{mol m}^{-2} \text{s}^{-1}$ | 2.4      | Sink capacity (eqn. (A7))                               |
| $K_c$            | 30     | Pa                                   | 2.1      | Michaelis constant, $\text{CO}_2$ (eqn. (A5))           |
| $K_o$            | 30     | kPa                                  | 1.2      | Inhibition constant of $\text{O}_2$ (eqn. (A5))         |
| $m$              | 9      |                                      |          | Slope parameter, B-B model (eqn. (1))                   |
| $m'$             | 31     |                                      |          | Slope parameter, J model                                |
| $m''$            | 167    |                                      |          | Slope parameter, L model                                |
| $\text{O}_2$     | 20.9   | kPa                                  |          | Partial pressure $\text{O}_2$                           |
| $P$              | $10^5$ | Pa                                   |          | Atmospheric pressure                                    |
| $R_d$            | 3      | $\mu\text{mol m}^{-2} \text{s}^{-1}$ | 2.0      | 'Day' respiration (eqn. (A10))                          |
| $R_{\text{sky}}$ | 340    | $\text{W m}^{-2}$                    |          | Long wave-length radiation from sky                     |
| $V_m$            | 200    | $\mu\text{mol m}^{-2} \text{s}^{-1}$ | 2.4      | Rubisco capacity (eqn. (A5))                            |
| $\alpha$         | 0.08   |                                      |          | Quantum efficiency (eqn. (A2))                          |
| $\beta$          | 0.98   |                                      |          | (Equation (A9))   |
| $\tau$           | 2600   |                                      | 0.57     | $\text{CO}_2/\text{O}_2$ -specificity ratio (eqn. (A3)) |
| $\theta$         | 0.95   |                                      |          | (Equation (A8))   |

<sup>1</sup>Symbols for variables used in the models.  $A$ , gross  $\text{CO}_2$  assimilation;  $A_n$ , net  $\text{CO}_2$  assimilation;  $c$ ,  $\text{CO}_2$  mole fraction (subscripts refer to the ambient air, a, leaf surface, s, and intercellular air spaces, i, and are used uniformly for all other variables);  $p$ ,  $\text{CO}_2$  partial pressure;  $C$ , a control coefficient;  $\mathcal{Q}$ , the vapor pressure gradient (water);  $e$ , the partial pressure of water vapor;  $e^*$ , the saturation vapor pressure of water at a specified temperature;  $h_s$ , relative humidity at the leaf surface;  $g_b$ , boundary-layer conductance;  $g_s$ , stomatal conductance;  $g_{\text{max}}$ , maximum stomatal conductance;  $J_c$ , the 'Rubisco-limited' capacity for  $\text{CO}_2$  fixation;  $J_E$ , the ' $Q_p$ -limited' capacity for  $A_n$ ;  $J_s$ , the 'sink-limited' capacity for  $A_n$ ;  $\Gamma_*$ , the  $p_i$  at which  $A=0$ ;  $\lambda E$ , the flux of latent heat carried by transpired water vapor;  $Q_p$ , the incident flux of photosynthetically active photons;  $R_s$ , the solar radiation;  $R_t$ , the total radiation;  $T_a$ , the temperature of the ambient air;  $T_l$ , the leaf temperature; RuBP, ribulose 1,5-bisphosphate.

*Energy balance sub-model*

Surface temperature is calculated from the explicit solution of a fourth-order approximation to the energy budget equation given by Paw U (1987). The response of  $e^*$  to temperature is described by a fourth-degree regression approximation of the Goff–Gratch equation, and regression constants are given by Paw U (1987).  $\lambda E$  and sensible heat flux are calculated from surface temperature and  $g_s$  using the usual molecular flux equations. The surface is considered to be horizontal, uniform and single sided. Vertical gradients through the surface in  $R_s$ , humidity, temperature and  $g_b$  are not considered.