# Canopy Reflectance, Photosynthesis, and Transpiration. III. A Reanalysis Using Improved Leaf Models and a New Canopy Integration Scheme.

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 $\mathbf{T}$  he theoretical analyses of Sellers (1985, 1987), which linked canopy spectral reflectance properties to (unstressed) photosynthetic rates and conductances, are critically reviewed and significant shortcomings are identified These are addressed in this article principally through the incorporation of a more sophisticated and realistic treatment of leaf physiological processes within a new canopy integration scheme It is assumed, based on ecophysiological observations and arguments, that leaf physiological properties vary throughout the plant canopy in response to the radiation-weighted timemean profile of photosynthetically active radiation (PAR) These modifications yield a simpler and more robust theoretical relationship between canopy biophysical rates (photosynthesis, conductance) and spectral vegetation indices (SVI) The results indicate that area-averaged SVI, as obtained from coarse resolution satellite sensors, may give good estimates of the area-integrals of photosynthesis and conductance even for spatially heterogenous (though physiologically uniform) vegetation covers

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#### **INTRODUCTION**

Over the last 10 years, there have been some important advances in our understanding of how leaves assimilate carbon and control the simultaneous loss of water vapor through their stomata Farquhar et al (1980) showed how a biochemical model of leaf CO<sub>2</sub> assimilation based on rate constants calculated from a consideration of the enzyme kinetics and electron transport properties of chloroplasts could yield a realistic description of photosynthesis for C<sub>3</sub> plants Collatz et al (1991), following Ball (1988), used this work and observations of stomatal conductance to construct a robust semiempirical model of leaf stomatal function which can reproduce the response of leaf conductance to changes in ambient temperature, humidity, CO<sub>2</sub> concentration, and assimilation rate A number of investigators (Field, 1983, Terashima and Inoue, 1985, Hirose and Werger, 1987, Gutschick and Wiegel, 1988, Farguhar, 1989, Evans, 1989a) explored consequences of the distribution of photosynthetic capacity in leaves and canopies with respect to light and developed criteria for identifying the distribution of any fixed total capacity that maximizes photosynthetic CO<sub>2</sub> assimilation

It would be useful to extend this knowledge

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of leaf-level processes up to the canopy scale (meters and kilometers) Among other applications, this could lead to the calculation of carbon fluxes and evapotranspiration rates on scales consistent with global biogeochemical cycle studies, see, for example, Tans et al (1990) To do this, it is necessary to quantify the relationships between canopy function and spectral signatures because satellite remote sensing offers the only practical means of continuously and consistently monitoring biospheric processes on a global scale

Sellers (1985, 1987) investigated methods of integrating simple leaf-level models of light scattering, light absorption, photosynthesis, and stomatal conductance over the depth of vegetation canopies His analysis explored a theoretical basis for analyzing the empirical connections between spectral vegetation indices (SVI) and important functional relationships that regulate canopy photosynthesis and transpiration A key result showed that for horizontally uniform (plane-parallel) canopies, there is a strong mechanistic basis for a correlation between the fraction of photosynthetically active radiation absorbed by the vegetation canopy (FPAR) and the associated simple ratio vegetation index (SR) (near-infrared reflectance divided by visible reflectance) The analysis also showed that the bulk canopy photosynthetic capacity and the maximum canopy conductance were near-linearly related to the SR However, the leaf physiological models used in the analysis of Sellers (1985, 1987) suffered from a number of shortcomings

- 1 The leaf  $CO_2$  assimilation and stomatal conductance models used simple empirical functions which are hard to parameterize
- 11 There was no linkage between stomatal function and leaf CO<sub>2</sub> assimilation
- 111 Leaf physiological properties (photosynthetic capacity, etc) were assumed to be invariant through the depth of the canopy

These issues are addressed in the analysis presented in this paper We demonstrate that the incorporation of a more sophisticated treatment of physiological processes results in a simpler and more robust relationship between canopy biophysical rates (photosynthesis, conductance) and spectral vegetation indices (SVI)

#### THEORETICAL BACKGROUND

#### Summary of the Analysis of Sellers (1985; 1987)

Many researchers have utilized combinations of spectral radiance observations acquired over vegetated surfaces by satellite or aircraft-mounted sensors as indicators of the density, health, or biomass of the vegetation. These empirical applications of remote sensing take advantage of the large difference between the light scattering properties of green leaves in the visible and nearinfrared wavelength intervals.

Sellers (1985, 1987) used a two-stream approximation model to describe radiative transfer within vegetation canopies. The equations obtained from the two-stream method may be used to calculate the hemispheric reflectance of a plant canopy as a function of a) the relative spectral response function of the sensor, b) the radiation field incident on the canopy, c) the soil or background reflectance, d) the scattering coefficients and geometric arrangement of the leaf elements, and e) the amount of vegetation present, as specified by the leaf area index, for example. A related procedure was used to calculate the profiles of radiation absorbed by leaves as a function of canopy depth

Sellers (1987) used these equations to show that for ideal conditions—uniform green canopy, dark underlying surface—the spectral vegetation indices (SVI) should be proportional to the nearinfrared reflectance,  $a_N$ , and to FPAR [referred to as APAR in Sellers (1985, 1987)] The most commonly used SVI are the simple ratio (SR) and the normalized difference (ND) vegetation indices, defined as

$$SR = \frac{a_{\Lambda}}{a_{\Lambda}},$$
 (1a)

$$ND = \frac{a_N - a_V}{a_N + a_V}$$
(1b)

where

## $a_{N}, a_{v}$ = hemispheric canopy reflectances for near-infrared and visible wavelength intervals, respectively (sensor-dependent)

Sellers (1987) showed that this useful relationship between the SVI and FPAR holds because the broad-band scattering coefficients of green leaves in the near-infrared ( $\omega_N$ ) and visible ( $\omega_V$ ) wavelength intervals are very different (Table 1), this difference is such that

$$\frac{\partial a_N}{\partial L_T} \propto \frac{\partial (\text{FPAR})}{\partial L_T}, \quad \text{for all values of } L_T, \quad (2)$$

where

 $L_{\tau}$  = total leaf area index

when the extinction coefficient for the flux of PAR or visible radiation (k) is roughly double the extinction coefficient for diffuse near-infrared flux  $(h_N)$  within the canopy, that is, when

$$k \simeq 2h_{\scriptscriptstyle N},\tag{3a}$$

which may be reexpressed as

$$\frac{G(\mu)}{\mu} (1 - \omega_v)^{1/2} = 2(-\omega_v)^{1/2}, \qquad (3b)$$

where

- $\omega_v, \omega_n$  = leaf scattering coefficients in the visible, near-infrared wavelength intervals, respectively (sensordependent),
  - k = extinction coefficient for direct(solar) beam flux within the canopy =  $[G(\mu) / \mu](1 - \omega_v)^{1/2}$ ,
  - $h_N = \text{extinction coefficient for diffuse}$ near-infrared flux within the canopy  $= (1 - \omega_N)^{1/2}$ ,

Table 1 Parameters Used To Calculate Leaf Photosynthesis and Conductance for the Models Used in Sellers (1985, 1987)

| Parameter          | Units   | Value           |
|--------------------|---|-----------------|
| Photosynthesis     |   |                 |
| $a_1$              | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup>                     | 52 0            |
| $\boldsymbol{b}_1$ | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup> , W m <sup>-2</sup> | 1380 0, 300 0   |
| Conductance        |   |                 |
| $a_2$              | µmol mol <sup>-1</sup> , J m <sup>-3</sup>                    | 1268 5, 13966 0 |
| $b_2$              | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup> , W m <sup>-2</sup> | 046,01          |
| $C_2$              | $(mol m^{-2} s^{-1})^{-1}$ , $s m^{-1}$                       | 0 55, 28 0      |
| Leaf properties    |   |                 |
| $G(\mu)$           | _   | 05              |
| ω                  | _   | 02              |
| $\omega_{N}$       | _   | 0 95            |
| Soil reflectance   |   |                 |
| ωs                 | -   | 01              |
| Solar angle        |   |                 |
| μ                  | $\cos^{-1}$   | 05              |

"Adapted from Charles-Edwards and Ludwig (1974) and Jarvis (1976), see Eq (1)

$$G(\mu)$$
 = relative projected area of leaves  
in direction  $\cos^{-1}\mu$ ,  
 $\mu$  = cosine of solar zenith angle

Simply put, Eq (2) holds because the nearinfrared reflectance  $a_N$  is proportional to *double* the pathlength of near-infrared radiation in the canopy  $[e^{-2h_NL_T}]$ , as this radiation must enter and leave the canopy, while FPAR is proportional to only the *one-way* penetration and absorption of PAR through the canopy  $[e^{-kL_T}]$  The two parameters,  $a_N$  and FPAR, will be proportional to each other if Eq (3) is satisfied

If the soil or background material underlying the canopy is relatively dark, so that

$$\frac{\partial a_v}{\partial L_T} \to 0,$$

we can write

$$\frac{\partial (\mathrm{SR})}{\partial L_{\mathrm{T}}} \propto \frac{\partial a_{\mathrm{N}}}{\partial L_{\mathrm{T}}} \tag{4}$$

It follows then that FPAR is proportional to SR

Figure 1a shows how the canopy visible and near-infrared reflectances,  $a_v$  and  $a_N$ , the simple ratio vegetation index SR, and FPAR vary with leaf area index for the (almost ideal) model canopy described in Table 1 Figure 1b illustrates how the nonlinear functions of leaf area index, SR and FPAR, shown in Figure 1a are almost linearly related to each other because of the relationship expressed in Eq. (3)

The analysis summarized above provides a theoretical foundation for the correlation between the SR and FPAR established by empirical analysis of remote sensing data, Eq (3) is normally a reasonable approximation for canopies composed of randomly distributed elements with identical reflectance properties overlying dark soils. It is important to note, however, that the system of equations given in Sellers (1987) provides a general basis for relating hemispheric reflectance measurements to canopy, leaf, and soil properties—regardless of whether the above approximation is true

Sellers (1985, 1987) also examined the relationship between canopy reflectance and the use of PAR by leaves for net photosynthesis  $A_n$  and the regulation of stomatal conductance  $g_s$  This is a more complex problem since these physiological

These parameters were obtained by curve fits to the PAR response functions as given by the Farquhar et al (1980) and Collatz et al (1991) models for stress-free (relative humidity = 1) conditions, see Figure 6e, f

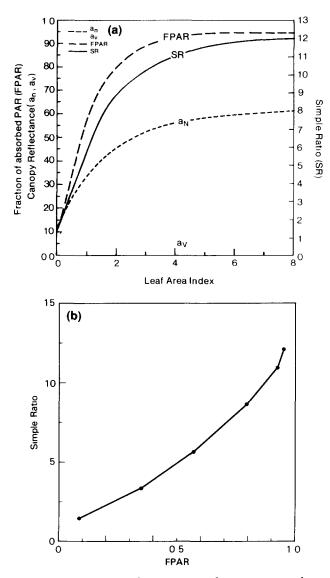


Figure 1 a) Variation of canopy optical parameters with leaf area index as calculated by the two-stream approximation model of Sellers (1985, 1987) with parameters taken from Table 1  $a_v$ ,  $a_v =$  visible and near infrared (hemispherically-integrated) reflectances, SR = simple ratio vegetation index, FPAR = fraction of PAR absorbed by the canopy b) Simple ratio vegetation index plotted against FPAR, replotted from Figure 1(a) Dots on the curve refer to values of leaf area index, 01, 05, 10, 20, 40, 80, reading from left to right

processes are influenced by several other variables (temperature, water vapor content of the air, water potential of the leaf, and the prior history of the leaves in the canopy) in addition to the absorbed flux of PAR

In the treatment of Sellers (1985, 1987), the models of Charles-Edwards and Ludwig (1974) and Jarvis (1976) were used to describe leaf photosynthesis and leaf stomatal conductance, respectively

$$A_{n} = \left[\frac{a_{1}\mathbf{F} \cdot \mathbf{n}}{b_{1} + \mathbf{F} \cdot \mathbf{n}}\right] [f(T)f(\varphi_{l})f(\delta e)], \qquad (5a)$$

$$g_{n} = \left[\frac{b_{2} + \mathbf{F} \cdot \mathbf{n}}{a_{2} + b_{2}c_{2} + c_{2}\mathbf{F} \cdot \mathbf{n}}\right] [f(T)f(\varphi_{l})f(\delta e)], \quad (5b)$$

where

$$A_{n} = \text{leaf photosynthesis}$$

$$(\mu \text{mol } \text{m}^{-2} \text{ s}^{-1})$$

$$\equiv P \text{ m Sellers (1985, 1987),}$$

$$a_{1},b_{1} = \text{species-dependent constants}$$

$$(\mu \text{mol } \text{m}^{-2} \text{ s}^{-1}, \text{ or } \text{W} \text{ m}^{-2}),$$

$$\mathbf{F} = (\text{vector) flux of PAR}$$

$$(\mu \text{mol } \text{m}^{-2} \text{ s}^{-2} \text{ s}^{-1} \text{ or } \text{W} \text{ m}^{-2}),$$

$$\mathbf{n} = \text{vector of leaf normal,}$$

$$f(T),f(\varphi_{l}),f(\delta e) = \text{adjustment factors to account}$$
for the effects of temperature, leaf water potential, and vapor pressure deficit stress,  

$$g_{s} = \text{leaf stomatal conductance for}$$

$$water vapor, (\text{mol } \text{m}^{-2} \text{ s}^{-1}),$$

$$a_{2},b_{2},c_{2} = \text{species-dependent constants}$$

$$(\text{mol } \text{mol}^{-1} \text{ or } \text{J} \text{ m}^{-3},$$

$$\text{mol } \text{m}^{-2} \text{ s}^{-1} \text{ or } \text{W} \text{ m}^{-2},$$

$$(\text{mol } \text{ m}^{-2} \text{ s}^{-1})^{-1} \text{ or } \text{ s} \text{ m}^{-1})$$

The formulation used in (5b) is consistent with the nomenclature used in Sellers (1985, 1987) It was assumed that all the leaves in the canopy respond identically to  $\mathbf{F}$  n The constants  $a_1$ ,  $b_1$ ,  $a_2$ ,  $b_2$ , and  $c_2$  can be determined from curve fits to data (see Table 1) The stress factors  $f(\mathbf{x})$  varv from unity, under optimal conditions, to zero when photosynthesis and transpiration are totally suppressed by adverse environmental conditions (see Jarvis, 1986, Sellers et al, 1989, Collatz et al, 1991)

The combination of the environmental stress factors was assumed to operate more or less uniformly throughout the canopy so that in estimating canopy photosynthesis and conductance it is only necessary to integrate Eqs (5) with respect to the variation of intercepted PAR,  $\mathbf{F}$  n, down through the canopy The canopy integral forms of (5) may then be written as

$$A_{\epsilon} = f(\Sigma) \int_{0}^{L_{\tau}} \left[ \frac{a_{1} \mathbf{F} \cdot \mathbf{n}}{b_{1} + \mathbf{F} \mathbf{n}} \right] dL, \qquad (6a)$$

$$g_{c} = f(\Sigma) \int_{0}^{L_{T}} \left[ \frac{b_{2} + \mathbf{F} \mathbf{n}}{a_{2} + b_{2}c_{2} + C_{2} \mathbf{F} \mathbf{n}} \right] dL, \quad (6b)$$

where

 $A_c = \text{canopy photosynthesis } (\mu \text{mol } m^{-2} \text{ s}^{-1}),$   $g_c = \text{canopy conductance}$   $(\text{mol } m^{-2} \text{ s}^{-1} \text{ or } m \text{ s}^{-1}),$  $(T) = (TT) (f_{c} + f_{c}) (f_{c} + f_{c})$ 

 $f(\Sigma) = f(T)f(\psi_l)f(\delta e)$ 

The PAR flux vertical component is assumed to be attenuated as it passes down through the canopy following the semiempirical expression of Goudriaan (1977)

$$F_{L} = F_{0}e^{-kL} \tag{7}$$

where

 $F_L$  = PAR flux at leaf area index L in the canopy (µmol m<sup>-2</sup> s<sup>-1</sup> or W m<sup>-2</sup>),  $F_0$  = PAR flux above the canopy (µmol m<sup>-2</sup> s<sup>-1</sup> or W m<sup>-2</sup>), L = cumulative leaf area index

Insertion of Eq. (7) into (6) allows evaluation of  $A_c$  and  $g_c$  (see Sellers, 1985, Tables 3 and 4) [Note The combination of (6) and (7) only accounts for the variation of PAR intensity with canopy depth, a mean leaf angle is assumed throughout Sellers (1985) explored the impact of this simplification on the calculation of the bulk canopy properties,  $A_c$  and  $g_c$  Full integrations over leaf angle and orientation were made prior to the integration with canopy depth, the difference between the results obtained with the full (leaf angles and orientations, canopy depth) and simplified (mean leaf angle / orientation, canopy depth) integrations was found to be practically negligible ]

We may now rewrite Eqs (6) as

$$A_c = A_c^* f(\Sigma), \tag{8a}$$

$$g_c = g_c^* f(\Sigma), \tag{8b}$$

where  $A_c^*$  and  $g_c^*$  are the integrated kernels of (6) and represent the canopy-scale values of the unstressed photosynthetic rate and conductance

Figures 2a and 2b show how  $A_n$ ,  $A_c^*$  and  $g_s$ ,  $g_c^*$  vary with incident PAR flux  $F_0$ , according to Eqs (5) and (6) (Here  $A_n$  and  $g_s$  refer to leaves at the very top of the canopy, fully exposed to the ambient PAR flux  $F_0$ )

Figures 2c and 2d show how  $A_c^*$  and  $g_c^*$  vary with total leaf area index For any given PAR flux, there is a diminishing increase in  $A_c^*$  and  $g_c^*$  for further increments of leaf area index as more and more of the canopy consists of shaded leaves which have progressively lower rates of  $A_n$  and  $g_s$ Now an inspection of the solutions to Eqs (6) or (8) [see Tables 3 and 4 in Sellers (1985)] shows that, for low values of the incident PAR flux  $F_0$ , the derivatives of  $A_c^*$  and  $g_c^*$  with respect to total canopy leaf area index  $L_T$  are proportional to the derivative of FPAR with  $L_T$  and to  $e^{-kL_T}$ 

$$\frac{\partial A_c^*}{\partial L_r}, \frac{\partial g_c^*}{\partial L_r} \propto e^{-kL_r}, \quad \frac{\partial (\text{FPAR})}{\partial L_r}, \quad \text{as} \ F_0 \to 0 \ (9)$$

Equation (9) holds because the PAR flux is used to drive photosynthesis and conductance as it is progressively absorbed through the canopy Figures 2e and 2f show  $A_c^*$  and  $g_c^*$  plotted against FPAR, and it can be seen that, for the lower values of  $F_0$ , Eq. (9) holds resonably well This is because, at low values of  $F_0$ , all the leaves in the canopy have approximately the same slope of A or g against  $F_0$  (see Figs 2a and 2b) At higher values of  $F_0$ , however, the leaves at the top of the canopy are saturated (no change in A or  $g_s$  with  $F_0$ ), but the leaves lower down are still below saturation, resulting in an increasingly nonlinear relationship between  $A_c^*$ ,  $g_c^*$ , and FPAR with increasing  $F_0$ This effect can be seen more clearly in Figures 2g and 2h, where  $A_c^*/F_0$  and  $g_c^*/F_0$  are plotted against FPAR The lines corresponding to the lowest values of  $F_0$  are the most nearly linear, degrading to more nonlinear forms as  $F_0$  increases

Comparing Eqs (2), (4), and (9), we have the chain of relationships

$$\frac{\partial A_c^*}{\partial L_r}, \frac{\partial g_c^*}{\partial L_r} \propto \frac{\partial (\text{FPAR})}{\partial L_T} \propto \frac{\partial a_N}{\partial L_r} \propto \frac{\partial (\text{SR})}{\partial L_r}, \quad \text{as} \quad F_0 \to 0$$
(10)

so that under the specified conditions—uniform canopy, dark underlying soil, low levels of  $F_0$ —  $A_c^*$  and  $g_c^*$  should be proportional to SR and other SVI However, on clear days  $F_0$  values typically reach 200–400 W m<sup>-2</sup> and so this approximation should be viewed with some caution Additionally, further nonlinear effects are induced when the vegetation within the field of view is clumped into spatially heterogenous units (see Fig 21 of Sellers (1985)]

This apparent nonlinearity of canopy function potentially degrades the prospects for realistic estimation of CO<sub>2</sub> assimilation using remote sens-

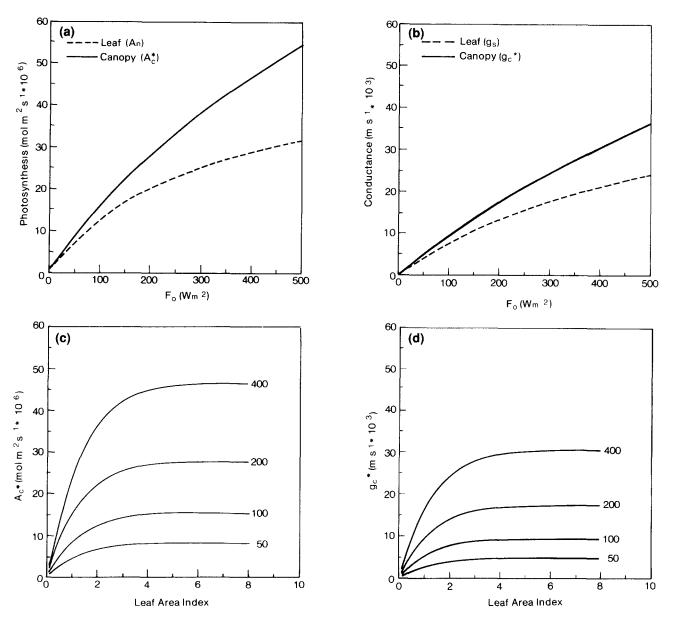


Figure 2 a,b) Unstressed leaf,  $A_n$  and  $g_n$ , and canopy,  $(A_c^* \text{ and } g_c^*)$ , photosynthesis and conductance as functions of incident PAR flux,  $F_0$ , calculated using Eqs (5) and (6) in text, parameters from Table 1 Total leaf area index for the canopy,  $L_r = 8$  c,d) Variation of unstressed canopy photosynthesis,  $A_c^*$ , and conductance,  $g_c^*$ , with leaf area index and PAR flux,  $F_0$ , calculated using Eqs (6) Values of  $F_0$  are marked on the curves

ing It results from the fact that, in the formulation of Sellers (1985, 1987), leaves at different levels in the canopy are saturating with respect to **F n** at different levels of  $F_0$  If photosynthetic capacity, as represented by the values of  $a_1$  and  $b_1$  in (5), is constant throughout the canopy, leaves near the top of the canopy would usually be completely light-saturated while leaves near the base would still be on the linear portion of the light-response curve and thus unsaturated This formulation certainly exaggerates any nonlinearity that occurs in nature, because leaves in shaded habits tend to have lower photosynthetic capacities and saturate at lower light levels (Bjorkman, 1981)

In the following sections, we extend the analysis of Sellers (1985) by replacing the empirical leaf models with more general, semimechanistic models of photosynthesis and stomatal conductance. In contrast to the uniform distribution of canopy properties with depth assumed above, we explore depth distributions of photosynthetic capacity and maximum stomatal conductance that

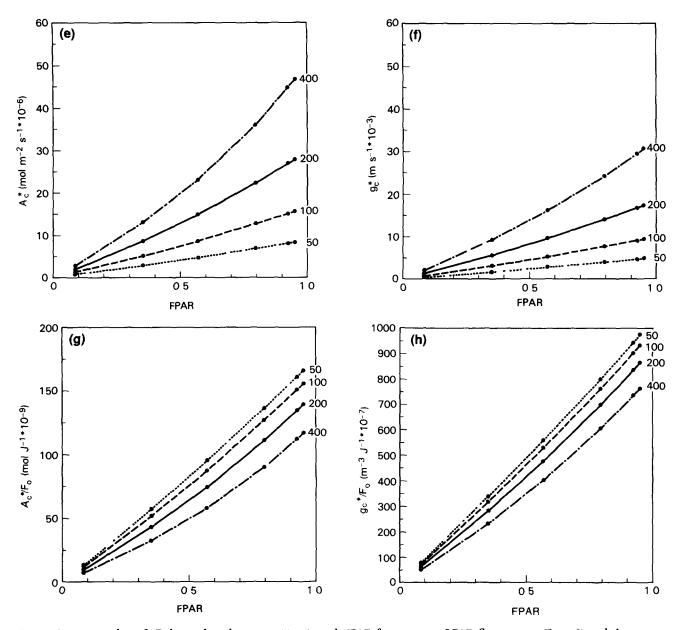


Figure 2 (continued) e.f. Relationships between  $A_c^*$ ,  $g_c^*$ , and FPAR for a range of PAR fluxes using Eqs (6) and the twostream approximation model described in Sellers (1985, 1987) g,h) Relationship between  $A_c^* / F_0$ ,  $g_c^* / F_0$  and FPAR,  $A_c^* / F_0$ and  $g_c^* / F_0$  can be considered as surrogates for  $\partial A_c^* / \partial F_0$  and  $\partial g_c^* / \partial F_0^*$ . The points on the curves refer to values of  $L_r = 0.1, 0.5, 1.0, 2.0, 4.0, 8.0$ , working from left to right. The numbers on each curve refer to values of  $F_0$  in W m<sup>-2</sup>. The canopy is assumed to be uniform and green, overlying a dark background. Parameter values are given in Table 1,  $f(\Sigma) = 1$ .

parallel the depth distribution of photosynthetically active radiation. This arrangement is supported by both an expanding base of empirical data and a number of theoretical assessments that identify an optimum depth distribution of photosynthetic capacity for maximizing canopy photosynthesis The results of this analysis are simpler and more general than those from the previous studies, and indicate that the relationships among SR, FPAR,  $A_c$ , and  $g_c$  should be almost independent of the spatial distribution of the vegetation

### **Improved Leaf Physiological Models**

Farquhar et al (1980) presented a biochemical model of leaf photosynthesis (including photorespiration), subsequently extended by von Caemmerer and Farquhar (1985) and other researchers, which describes CO<sub>2</sub> assimilation, A, as rate-limited by enzyme kinetics, specifically the amount and cycle time of the carboxylating enzyme Rubisco, and electron transport, which is a function of incident PAR and the efficiency of the leaf's lightintercepting apparatus (chlorophyll) In a recent version of the model (Collatz et al, 1991), A is given by  $A = \min(w_c, w_e, w_s)$ , where  $w_c, w_e, w_s$  are functions which desribe the assimilation rates as limited by the efficiency of the photosynthetic enzyme system, the amount of PAR captured by the leaf chlorophyll and the capacity of the leaf to export or utilize the products of photosynthesis, respectively

The ribulose-bisphosphate (RuBP) carboxylaseoxygenase enzyme (Rubisco) limiting rate on assimilation,  $w_c$ , is given by

$$w_{c} = V_{m} \left[ \frac{C_{c} - \Gamma^{*}}{C_{c} + K_{c} (1 + O_{2} / K_{o})} \right]$$
(11)

where

- $w_c$  = Rubisco-limited rate of assimilation ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>),
- $V_m$  = maximum catalytic capacity of Rubisco ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>)
- $C_i$  = concentration of CO<sub>2</sub> in leaf interior (Pa),
- $O_2 =$ partial pressure  $O_2$  in leaf interior (Pa),
- $\Gamma^* = CO_2$  compensation point (Pa), *K*<sub>c</sub> = Michaelis-Menten constant for

$$CO_2$$
, (Pa),  
 $K_o =$  inhibition constant for  $O_2$  (Pa)

 $V_m$  is given by the product of  $V_{max}$  and a temperature-dependent function [see Appendix of Collatz et al (1991) and Table 2]  $V_{max}$  is a property of the leaf (or chloroplast) and is proportional to the Rubisco reserves of the leaf (or chloroplast) and thus its nitrogen content  $\Gamma^*$ ,  $K_c$ , and  $K_o$  are all functions of temperature [see Appendix of Collatz et al (1991) and Table 2 of this article]

The light-limited rate of assimilation,  $w_e$ , is given by

$$\boldsymbol{u}_{\iota} = (\mathbf{F} \ \mathbf{n}) \boldsymbol{\varepsilon} (1 - \boldsymbol{\omega}_{\iota}) \left[ \frac{C_{\iota} - \Gamma^{*}}{C_{\iota} + 2\Gamma^{*}} \right]$$
(12)

where

- $w_i =$ light-limited rate of assimilation ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>),
- $\varepsilon$  = intrinsic quantum efficiency for CO<sub>2</sub> uptake, ( $\mu$ mol  $\mu$ mol<sup>-1</sup> or  $\mu$ mol J<sup>-1</sup>)

A third limiting rate has been defined by Collatz et al (1991)  $w_{,}$  is the capacity for the export or utilization of the products of photosynthesis and is estimated by Collatz et al (1991) to be

$$u_{s} = V_{m}/2 \tag{13}$$

The simplest way to proceed is to assume that the assimilation rate is the minimum of  $w_i$ ,  $u_i$ , and  $w_i$  (cf Farquhar et al , 1980) However, observations indicate that the transition from one limiting rate to another is not instantaneous and that coupling between the three processes leads to smooth curves rather than superpositioning of straight lines Collatz et al (1991) describe this effect by combining the rate terms into two quadratic equations, which are then solved for their smaller roots

$$\theta w_p^2 - u_p(u_{\epsilon} + u_{\epsilon}) + w_{\epsilon} w_{\epsilon} = 0, \qquad (14a)$$

$$\beta A^2 - A(w_p + u_s) + w_p w_s = 0,$$
 (14b)

where

A = assimilation rate ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>),  $\theta, \beta$  = coupling coefficients,  $w_p$  = "smoothed" minimum of  $w_c$  and  $w_c$  ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>)

The coefficients  $\theta$  and  $\beta$  can theoretically range from 1 (no coupling effects) to 0. In nature, these coefficients assume values on the order of 0.8 to 0.99 (see Collatz et al., 1990). Figure 6a shows an example of a sharp transition from  $u_{\iota}$ to  $w_c$  ( $\theta$ ,  $\beta = 1$ ), Figure 6e shows a more gradual transition ( $\theta$ ,  $\beta = 0.8$ ).

Net assimilation  $A_n$  is then given by

$$A_n = A - R_d, \tag{15}$$

where

 $R_d$  = leaf respiration rate ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>)

Collatz et al (1991) scaled  $R_d$  to the leaf carboxylase content by

$$R_d = 0 \ 015 V_m \tag{16}$$

| A Forcing Variables   |  |                          |  |  |  |  |
|-----------------------|--|--------------------------|--|--|--|--|
| Symbol                | Variable   | Values                   | Units  |  |  |  |
| Ca                    | CO <sub>2</sub> concentration in CAS <sup>†</sup>          | 34                       | Pa   |  |  |  |
| $F_0$                 | PAR flux above canopy                                      | 50, 100, 200, 400        | $W m^{-2}$   |  |  |  |
| -                     |  | 230, 460, 920, 1840      | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup>                |  |  |  |
| gь                    | Leaf boundary layer conductance (H <sub>2</sub> O)         | 0 04                     | $m s^{-1}$   |  |  |  |
| $\tilde{h}_a$         | Relative humidity of CAS                                   | 01, 025, 05, 075         |  |  |  |  |
| <b>O</b> <sub>2</sub> | O <sub>2</sub> concentration in CAS <sup>†</sup>           | 20,900                   | Ра   |  |  |  |
| р                     | Atmospheric pressure <sup>†</sup>                          | $1\ 013 \times 10^{5}$   | Pa   |  |  |  |
| T <sub>s</sub>        | Leaf temperature   | 310                      | K  |  |  |  |
| μ                     | Cosine of incident angle of PAR flux                       | 0 25, 0 5, 0 75, 1 0     |  |  |  |  |
|                       |  | arameters                |  |  |  |  |
| b                     | Minimum stomatal conductance <sup>†</sup>                  | 0 01                     | mol m <sup>-2</sup> s <sup>-1</sup>                      |  |  |  |
| $G(\mu)$              | Leaf angle distribution function                           | 0 5                      | _  |  |  |  |
| ÷ /                   | (replaces n)   |                          |  |  |  |  |
| Kc                    | Michaelis-Menten coefficient for CO2 <sup>†</sup>          | 30*2 1 <sup>o</sup>      | Ра   |  |  |  |
| K <sub>o</sub>        | Inhibition coefficient for $O_2^{\dagger}$                 | 30,000*1 2 <sup>ot</sup> | Pa   |  |  |  |
| Lr                    | Total leaf area index                                      | 0 1, 0 5, 1 0,           | $m^2 m^{-2}$   |  |  |  |
|                       |  | 20,40,80                 |  |  |  |  |
| m                     | Slope parameter <sup>†</sup>                               | 90                       | _  |  |  |  |
| $Q_t$                 | $Q_{10}$ temperature coefficient <sup>†</sup>              | $(T_s - 298) / 10$       |  |  |  |  |
| Š                     | $CO_2 / O_2$ specificity <sup>†</sup>                      | 2600*0 57°               |  |  |  |  |
| $V_m$                 | Maximum leaf catalytic capacity at $T_s$                   | $V_{\max}*2^{o_t}$       | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup>                |  |  |  |
| V <sub>max</sub>      | Maximum leaf catalytic capacity at 298 K                   | 200                      | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup>                |  |  |  |
| $V_{\max_0}$          | $V_{\rm max}$ for top leaves <sup>†</sup>                  | 200                      | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup>                |  |  |  |
| Г*                    | CO <sub>2</sub> photocompensation point <sup>†</sup>       | $O_2 / (2S)$             | Ра   |  |  |  |
| ε                     | Quantum efficiency for CO <sub>2</sub> uptake <sup>†</sup> | 0.08                     | mol mol <sup>-1</sup>                                    |  |  |  |
| ω                     | Leaf scattering coefficient for PAR                        | 02                       | _  |  |  |  |
| θ, β                  | "Linkage" coefficients between                             | 1 0, 1 0,                | _  |  |  |  |
|                       | $w_e, w_c, w_s$  | 08,08                    |  |  |  |  |
|                       |  | d Variables              |  |  |  |  |
| A                     | Assimilation rate  | _                        | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup>                |  |  |  |
| C,                    | Leaf interior CO <sub>2</sub> concentration                | _                        | Pa   |  |  |  |
| C,                    | Leaf surface CO <sub>2</sub> concentration                 | -                        | Pa   |  |  |  |
| E                     | Transpiration rate   | _                        | mol m <sup>-2</sup> s <sup>-1</sup> or m s <sup>-1</sup> |  |  |  |
| g,                    | Leaf stomatal conductance (H2O)                            | _                        | mol m <sup>-2</sup> s <sup>-1</sup> or m s <sup>-1</sup> |  |  |  |
| ĥ,                    | Leaf surface specific humidity                             | _                        | -  |  |  |  |
| R <sub>d</sub>        | Respiration rate   | _                        | $\mu$ mol m <sup>-2</sup> s <sup>-1</sup>                |  |  |  |

| Table 2 | Forcing Variables, Leaf Parameters, and Derived Output Variables for the Coupled | l |
|---------|--|---|
|         | Photosynthesis Leaf Physiology Models  |   |

Collatz et al (1991) went on to incorporate the above photosynthesis model with the Ball (1988) semiempirical model for leaf stomatal resistance

$$g_s = m \frac{A_n}{C_s} h_s p + b, \qquad (17)$$

where

- $g_s$  = stomatal conductance for water vapor (mol m<sup>-2</sup> s<sup>-1</sup> or m s<sup>-1</sup>), m = coefficient from observations  $\approx$  9 for C<sub>3</sub> plants,
- b = coefficient from observations  $\approx 0.01 \text{ for } C_3 \text{ plants}$ (mol m<sup>-2</sup> s<sup>-1</sup> or m s<sup>-1</sup>),  $h_s = \text{relative humidity at leaf}$ surface,  $C_s = CO_2$  concentration at leaf surface (Pa), p = atmospheric pressure (Pa),  $p_s = \text{standard atmospheric pressure}$   $= 1.013 \times 10^5$  (Pa),  $T_f = \text{freezing temperature} = 273.16 \text{ K},$  $g_s \text{ (m s^{-1})} = 0.0244 \frac{T}{T_f p} g_s \text{ (mol m}^{-2} \text{ s}^{-1})$

<sup>&</sup>lt;sup>a</sup> Values of forcing variables and leaf parameters used to test integration schemes were taken from Collatz et al (1991) when indicated by † CAS stands for canopy air space

The leaf surface environment variables, h, and C,, are given by (see Fig. 3)

$$(H_s - H_a)g_b = (H_i - H_s)g_s = E$$
 (18a)

We also see from Figure 3 that

$$h_{s} = \left[\frac{H_{s}}{H_{t}}\right], \tag{18b}$$

$$\frac{(C_a - C_s)}{p} \frac{g_b}{1 \ 4} = \frac{(C_s - C_l)}{p} \frac{g_s}{1 \ 6} = A_n, \qquad (18c)$$

where

- $H_a H_s, H_i$  = water vapor concentration in the external air, at the leaf surface and inside the leaf, respectively (mol mol<sup>-1</sup>),
  - $g_b = \text{leaf boundary layer conductance for}$ water vapor (mol m<sup>-2</sup> s<sup>-1</sup> or m s<sup>-1</sup>),
  - E = transpiration rate (mol m<sup>-2</sup> s<sup>-1</sup>),
  - $C_a, C_s = \text{carbon dioxide concentration in the external air and at the leaf surface, respectively (Pa),}$
- 1 4, 1 6 = factors to account for different diffusivities of  $H_2O$  and  $CO_2$ in the leaf boundary layer and stomatal pores, respectively

The system is then closed by calculating the  $CO_2$  concentration of the leaf interior

$$C_i = C_s - \frac{1}{g_g} \frac{6A_n}{g_g} p \tag{19}$$

Figure 3 Coupled stomatal-photosynthesis models as described by Collatz et al (1991) for canopy photosynthesis and transpiration System shows pathways for heat,  $CO_2$  and water vapor flux

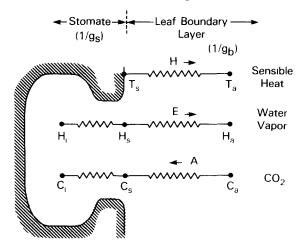


Figure 3 shows how the fluxes of water vapor, temperature, and heat from a leaf to the external air are assumed to behave in this model Figure 4 shows how the system of equations is solved in practice Table 2 lists the required forcing variables (environmental conditions), leaf parameters, and the derived (output) variables given by the solution of the equation set described in this section and depicted in Figure 4

## The Scaling of Leaf Physiology to Canopy Position

Leaves in shaded microsites have lower photosynthetic capacities and maximum stomatal conductances than leaves in exposed microsites (Bjorkman, 1981) This extremely general pattern applies to plants grown in controlled environments with high and low light (Bjorkman and Holmgren, 1963, Evans, 1989a), to lower and upper leaves on a single plant (Field, 1983, Hirose and Werger, 1987), and to mixed species assemblages with shade-tolerant understory species beneath a canopy of different species (Bazzaz, 1979, Schimel et al , 1991) Species differ greatly in their phenotypic plasticity (Seemann et al , 1987,

Figure 4 Iterative solution method for the

photosynthesis-stomatal conductance model, after Collatz et al. (1991) equation set

Equation  

$$V_{m} = f(V_{max} = T)$$
(16)  

$$R_{d} = f(V_{m})$$

$$K_{c} = K_{0} = f(T)$$

$$\Gamma^{*} = f(T, O_{2})$$
Estimate  $C_{i}$ 
(11)  

$$w_{c} = f(C_{i}, V_{m}, \Gamma^{*}, K_{0}, K_{c})$$
(12)  

$$w_{e} = f(C_{i}, F \cdot n, \epsilon, \omega = \Gamma^{*})$$
Adjust  
(13)  

$$w_{s} = f(V_{m})$$

$$C_{i}$$
(14 15)  

$$A_{n} = f(w_{c} = w_{e}, w_{s} = \Theta, \beta) - R_{d}$$
(17,18)  

$$g_{s} = h_{s} = C_{s} = f(A_{n} = m, b, g_{b} = C_{a})$$
(19)  
New estimate,  

$$C_{i} = f(C_{s} = A_{n}, g_{s})$$

$$C_{i} = f(C_{s} = A_{n}, g_{s})$$

$$Fxit$$

Evans, 1989a), in response to growth in low or high-light habitats (Bjorkman and Holmgren, 1963), and in the extent to which stress factors other than light depress photosynthetic capacity (Mooney, 1972) All of this intrinsic complexity is the raw material from which ecological and evolutionary factors shape plant canopies, with the result that the depth-distribution of photosynthetic capacity in real canopies tends to be quite predictable

Why does the shaping of plant canopies of ecological and evolutionary factors constrain the response of photosynthetic capacity to light availability<sup>2</sup> Three factors appear to be critical First, the basic mechanisms of photosynthesis are essentially identical in all C3 plants (more than 85% of all plant species), and differences in capacity are largely driven by differences in investment in the biochemical machinery of photosynthesis The strong, linear relationship between photosynthetic capacity and leaf nitrogen (Field and Mooney, 1986, Evans, 1989b) attests to this linkage between investment and capacity Second, the investments required to support photosynthesis are expensive Nitrogen nutrition alone can consume from 20% to 45% of the carbon fixed in photosynthesis (Chapin et al, 1987) Third, any plant that invests too much photosynthetic capacity in any leaf should be at a competitive disadvantage to a plant that matches investments to local resource (especially light) availability For a single plant with leaves distributed through a canopy, the criterion for the most efficient distribution of capacity is given by the solution to the isoperimetric problem in dynamic control theory (see Intriligator, 1971, Bloom et al, 1985) Specifically, if photosynthetic capacity is limited by some guantity Z that is expensive for the plant to acquire, then the returns on any fixed investment in Z are greatest when

$$\frac{\partial A_{\rm int}}{\partial Z} = \lambda, \qquad (20)$$

where  $A_{\rm int}$  is photosynthesis integrated over an appropriate spatial or temporal scale and  $\lambda$  is an undefined Lagrangian multiplier. In the analyses to date, Z has been considered as transpiration in a single leaf (Cowan and Farquhar, 1977), leaf nitrogen in a canopy (Field, 1983), leaf mass in a canopy (Gutschick and Wiegel, 1988), and photosynthetic capacity in a leaf (Farquhar, 1989)  $A_{\rm int}$  is typically the instantaneous value of photosynthesis for single leaf analyses but the single-leaf daily integral for canopy analyses. If the condition in Eq (20) holds for instantaneous values, it also holds for daily integrals. The problem of optimal resource allocation has no general solution when a canopy is composed of many plants or species, but the single plant solution may be approximately correct, as long as all plants tap a common pool of below-ground resources and construct tissues of similar composition and cost

PAR is also expensive to acquire, because plants cannot capture it without investing in leaves, stems, and branches Formally, the problem is identical to those already solved As long as

$$\frac{\partial^2 A}{\partial \left[ (\mathbf{F} \cdot \mathbf{n})(1 - \boldsymbol{\omega}_v) \right]^2} < 0$$
 (21)

for all A, then we can also write

$$\frac{\partial A}{\partial [(\mathbf{F} \cdot \mathbf{n})(1 - \omega_v)]} = \lambda_1, \qquad (22a)$$

which leads to

$$\frac{\partial A_c}{\partial (\text{FPAR})} = \lambda_2, \qquad (22b)$$

where the value of  $\lambda$  is different for each quantity (water, nitrogen, mass, or PAR) and also depends on the level of each quantity invested as well as on the status of the environmental variables that influence photosynthesis and transpiration

Equation (22) implies that, at all levels in the canopy, leaves begin to saturate with respect to  $(\mathbf{F} \mathbf{n})(1-\omega_v)$  at the same  $F_0$  In fact, the light response curves for leaves at all levels of the canopy should be scaled versions of a single response, with a scaling factor proportional to  $[(\mathbf{F} \mathbf{n})(1-\omega_v)]/F_0$  As long as dark respiration is the same at all levels in the canopy or is proportional to  $V_{\text{max}}$  [as in this analysis, see Eq (16)], then the light saturated photosynthetic rate, or  $A_{\text{max}}$ , should also be proportional to  $[(\mathbf{F} \mathbf{n})]/(1-\omega_v)]/F_0$  or to  $(\mathbf{F} \mathbf{n})(1-\omega_v)$ 

At any level in the canopy, the value of  $[(\mathbf{F} \mathbf{n})(1-\omega_v)]/F_0$  changes during the day, on a time scale that changes in  $A_{\text{max}}$  are unlikely to track Again, for optimum efficiency, adjustments in  $A_{\text{max}}$  are likely to follow a moving time average of the light regime An expanding body of empirical data, including the results of Field (1983),

Walters and Field (1987), and Pons et al (1990), directly supports the prediction that  $A_{max}$  for each leaf should be proportional to the time-averaged or integrated PAR that that leaf experiences Additional studies, including DeJong and Doyle (1985), Hirose and Werger (1987), Hirose et al (1989), Kittel et al (1990), and Schimel et al (1991), reported that leaf nitrogen, a strong correlate of  $A_{max}$ , parallels integrated PAR These studies extend the generality of the relationship, because they involve a multispecies prairie (Schimel et al, 1991) and single-species swards in which the leaf-age gradient places the youngest leaves at either the top of the canopy (Hirose and Werger, 1987) or the bottom (Hirose et al., 1989) Over a broad range of species and ecosystems, and with few exceptions (e.g., Leverenz and Jarvis, 1980), the general pattern matches the prediction from theory  $-A_{max}$  and leaf nitrogen [N] - should scale with the time-integral of the absorbed local PAR

These arguments and Eq (22) suggest that the profile of leaf nitrogen, [N], and  $V_{max}$  down through the canopy should follow some time average of the PAR flux, most likely the radiationweighted, time-mean  $\overline{\mathbf{F}}$  However, while this paradigm describes the relative distribution of [N], it does not say anything about the absolute values of [N] and  $V_{max}$  within the canopy, which are more likely to be constrained by the overall availability of nutrients within the plants environment. For optimal efficiency, canopies can be expected to allocate [N] such that for a specified light regime ( $\mathbf{F}$  n) operating over a period T on the order of several days to a few weeks

$$U_1 \int_0^{\tau} A \ dt + U_2 \int_0^{\tau} \frac{A}{[N]} \ dt, \qquad (23)$$

#### is maximized for all L

 $U_1$  and  $U_2$  are cost-benefit weighting factors which would be largely determined by the local availability of nitrogen In a nitrogen-rich environment, where there is little cost in maintaining high values of  $V_{max}$ ,  $U_1$  should be much greater than  $U_2$  and so maximizing photosynthesis [first term in Eq. (23)] would have a higher priority than maximizing efficiency [second term in Eq. (23)] Conversely, in nutrient-poor environments, we can expect  $U_2$  to be greater than  $U_1$  Whatever the conditions, the value of  $V_{max}$  arrived at from (23) can be associated with a reference value of PAR flux,  $\tilde{\mathbf{F}}$ , which light-saturates the leaves, that is,  $\mathbf{\tilde{F}} = \mathbf{F}$  when  $w_{\epsilon} = w_{\epsilon}$  Under most conditions, when  $U_1 >> U_2$ , this reference value of  $\mathbf{\tilde{F}}$  would be close to, if not equal to, the radiation-weighted time-mean value of  $\mathbf{F}$ ,  $\mathbf{\overline{F}}$ , for smoothly varying fluxes for simplicity, we shall assume that  $\mathbf{\tilde{F}} \simeq \mathbf{\overline{F}}$ from now on

## INTEGRATING THE COUPLED LEAF STOMATAL-PHOTOSYNTHESIS MODEL OVER THE CANOPY

From the analysis and observations discussed in the previous section, we propose that the profiles of leaf nitrogen, [N],  $V_{\text{max}}$ , and hence  $V_m$  within the plant canopy are distributed according to the radiation-weighted time-mean profile of PAR

$$V_m = V_{m_0} \overline{f(L)} \tag{24a}$$

where

$$V_{m_0}$$
 = maximum, that is, "top" leaf, value  
of  $V_m$  in the canopy ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>)  
= product of  $V_{max_0}$  and a temperature  
inhibition function  
(see Table 2).

$$\overline{f(L)} = \text{time-averaged variation of PAR} \\ \text{flux with LAI [e g, e^{-kL} Eq (7)]}$$

Also, for most conditions where nutrients are not in drastically short supply, that is,  $U_1 >> U_2$  in Eq (23), the value of  $V_{m_0}$  will be determined by the radiation-weighted time-mean flux of PAR,  $\overline{F}_0$ , at the top of the canopy, that is,

$$V_{m_0} = \text{function of } (w_{\epsilon} = w_{\epsilon})$$
(24b)  
when  $\mathbf{F}_0 = \overline{\mathbf{F}}_0$ 

To obtain  $V_{m_0}$  as defined in (24b),  $\mathbf{F}_0$  is inserted into (12) and the derived value of  $w_{\epsilon}$  used in place of  $w_{\epsilon}$  in (11) to calculate  $V_m$  (in this case  $V_{m_0}$ )

The overbar on f(L) in (24a) denotes "radiation-weighted time-mean value" Using the example of Eq. (7) to describe f(L), we then have

$$\overline{f(L)} = e^{-\overline{k}t}, \qquad (25a)$$

$$\overline{\mathbf{F}} = \overline{\mathbf{F}}_0 \,\overline{f(L)},\tag{25b}$$

$$\overline{k} = [G(\mu) / \mu] (1 - w_v)^{1/2}$$
 (25c)

Note that Eq. (25) also implies a time-mean zenith angle  $\overline{\mu}$  for the flux  $\overline{F}$ 

The instantaneous light-limited value of photosynthesis,  $w_e$ , is given by Eq. (12) A full treatment of canopy photosynthesis would take into account the entire range of leaf angles and orientations at every level on the canopy in addition to carrying out the top-to-bottom integration (PAR attenuation as described by Eq. (7)), in other words, the **F** n term would be subjected to full azimuth and zenith integrations. However, as discussed in Sellers (1985), the adoption of a single mean leaf projection  $G(\mu)$  in place of n gives very similar results numerically. The expression (**F** n) in (12) can therefore be replaced by a simpler function

$$\mathbf{F} \cdot \mathbf{n} = F \frac{G(\mu)}{\mu}$$
$$= F_0 \frac{G(\mu)}{\mu} f(L)$$
(26)

We can now construct a canopy-scale model of photosynthesis Equation (24) is combined with Eq (11) to give an expression for the variation of  $w_c$  with canopy depth

$$w_c = a_0 \overline{f(L)}, \quad a_0 = V_{m_0} \left[ \frac{C_i - \Gamma^*}{C_i + K_c (1 + O_2 / K_o)} \right]$$
 (27a)

where

 $V_{m_0}$  = mean value of  $V_m$  for the ensemble of leaves at the top of the canopy ( $\mu$ mol m<sup>-2</sup> s<sup>-1</sup>)

Equation (26) is combined with (12) to give an equivalent expression for  $w_e$ 

$$w_{e} = b_{0}f(L), \quad b_{0} = \left(\mathbf{F}_{0}\frac{G(\mu)}{\mu}\right)\varepsilon(1-\omega_{v})\left[\frac{C_{i}-\Gamma^{*}}{C_{i}+2\Gamma^{*}}\right]$$
(27b)

The f(L) terms in (27a) and (27b) are different

- 1 In (27a), the profile of  $V_m$  within the canopy is given by the product of  $V_m$  at the top of the canopy,  $V_{m_0}$ , and the radiation-weighted *time-mean* profile of PAR flux down through the canopy,  $\overline{f(L)}$  [ $V_{m_0}$  is related to  $\overline{\mathbf{F}}_0$ , the reference or *time-mean* (overbar) PAR flux as given by (24a)]
- 11 In (27b),  $w_e$  varies with f(L), the *instanta-neous* (no overbar) attenuation function for PAR down through a canopy, and  $\mathbf{F}_0$ , both of which vary with solar angle and cloud conditions

In reality, leaves within a canopy are subject to variations in leaf surface relative humidity  $(h_s)$ , leaf surface  $CO_2$  concentration ( $C_s$ ), boundary layer conductance  $(g_b)$ , leaf temperature  $(T_s)$  and a profile of temperature  $(T_a)$ , water vapor concentration  $(H_a)$ , and carbon dioxide concentration  $(C_a)$ in the canopy air space The resulting resistance network for this "real" situation is shown in Figure 5a Complete integration of Eq. (27) over a canopy is fairly complex, requiring a numerically solved multilayer model of the type described in Sellers and Lockwood (1981) or Goudriaan (1977) In most cases, however, it can be assumed that variations in leaf temperature  $(T_s)$  and the canopy air space profile gradients of  $T_a$ ,  $H_a$ , and  $C_a$  are small so that the resistance network can be simplified to that shown in Figure 5b, where  $T_s$ ,  $T_a$ ,  $e_a$ , and  $C_a$  are assumed to be invariant with depth Sensitivity studies by Sellers and Lockwood (1981) indicated that, for the case of modeled transpiration from dry canopies, the difference between results produced by the integration schemes represented by Figures 5a and 5b is relatively small

Following Figure 5b, the solution of the equation set for the entire canopy is now a relatively simple matter of numerical integration, more or less following the procedure used in Figure 4 Typical values of the forcing variables and leaf parameters listed in Table 2 were extracted from Collatz et al (1991) to construct a model copy It will be remembered that  $V_m$  is given by the product of a leaf physiological property,  $V_{\text{max}}$ , and a temperature function (see Table 2) In this case, the value of  $V_{\text{max}0}$  was taken to be equal to the value of  $V_{\text{max}}$  in Collatz et al (1991) and the profile of  $V_{\text{max}}$  was given by Eq. (24) For this worked example  $\overline{\mu}$ ,  $G(\overline{\mu}) = 0.5$  This corresponds to a canopy of spherically distributed leaves exposed to a time-mean flux of  $F_0$  with direction of  $\overline{\mu} = 0.5$ (solar zenith angle =  $60^\circ$ ) In Table 2, the quantum efficiency coefficient  $\varepsilon$  is assumed to be constant with canopy depth, following the observations of Ehleringer and Bjorkman (1977) A maximum leaf area index  $L_r$  of 8 was chosen for the study which represents a near maximum for normal broad-leaf conditions and provides a severe test of the integration schemes over the depth-varying PAR regime In this and all subsequent calculations, the vegetation is assumed to be free of soil moisture stress

Figure 6 shows the light response curves for leaves at different levels in the model canopy note in Figures 6c and 6d how the leaves all

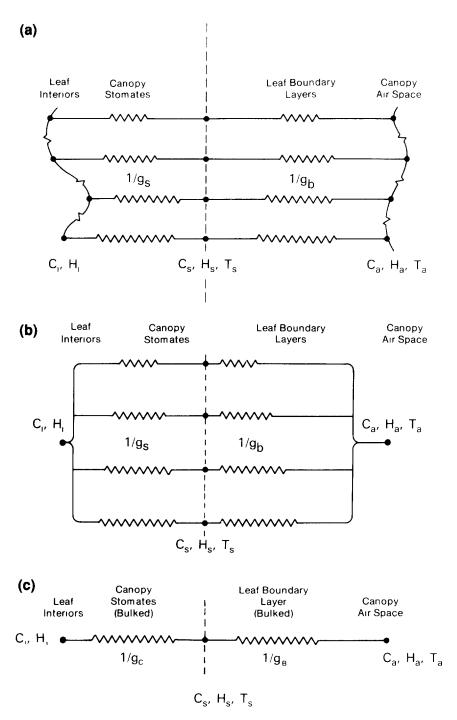


Figure 5 Resistance network corresponding to different canopy integration schemes a) Full scheme with developed profiles of  $T_a$ ,  $H_a$ ,  $C_a$ , and T, b) Restricted numerical scheme with  $T_a$ ,  $H_a$ ,  $C_a$ , and T, invariant with canopy depth c) Bulk (semianalytical) scheme with  $T_a$ ,  $H_a$ ,  $C_a$ , and  $T_s$  invariant with depth and bulk values of  $C_i$ ,  $H_s$ , and  $C_s$ . In a) and b), **F** has direction  $\mu$ , where  $\mu$  is not necessarily  $\overline{\mu}$ . In c),  $\mu = \overline{\mu}$ 

saturate at one value of  $\mathbf{F}_0$ , thus satisfying Eq (22) [The curves for the uppermost leaves in Figures 6e and 6f ( $\theta$ ,  $\beta = 0.8$ ) were used to fit Eqs (5) and derive the parameters  $a_1$ ,  $b_1$ ,  $a_2$ ,  $b_2$ , and  $c_2$  listed in Table 1 and used in Fig. 2]

Figures 7a-f show some results from simulations based on numerically integrating (27) over the canopy-80 layers were used Note how the behavior of the complete canopy follows that of the top leaves when  $h_s \rightarrow 1$  (compare Figs 6 and 7) Figure 8 shows how various biophysical states and rates can vary with depth in the model canopy the profiles of  $C_i$ ,  $h_s$ ,  $g_s$ , and A are fairly well developed In Figure 9, the effect of PAR intensity and incidence angle is shown, except for the extreme and physically implausible cause of high

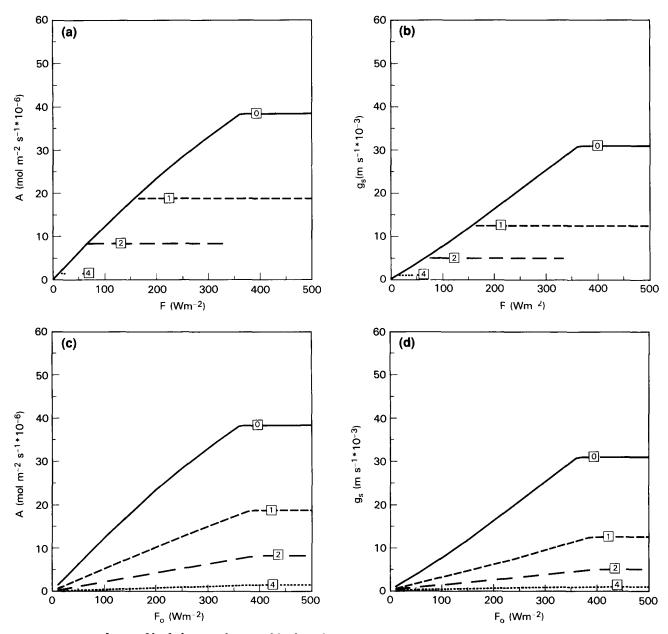


Figure 6 Dependence of leaf photosynthesis and leaf conductance on PAR as calculated using the coupled stomatal-photosynthesis model (Fig 4) In all cases, relative humidity in the canopy air space was held at unity,  $h_s \rightarrow 1$ , and leaf respiration  $R_d$  was set to zero. The numbers in boxes refer to leaf position in terms of cumulative leaf area index a,b) A and  $g_s$  versus the mean value of incident, PAR,  $F_s$  for leaves at different depths in the canopy, L = 0 refers to "top" leaves  $\theta$ ,  $\beta = 1.0$  c,d). Same as a) and b) except that all leaf responses are plotted against  $\mathbf{F}_0$ , note how all leaves saturate at roughly the same value of  $\mathbf{F}_0$ ,  $\theta, \beta = 1.0$ .

PAR fluxes at glancing angles (i.e., to the left of the numerical solution maxima), the canopy process rates show little variation with  $\mu$ 

Figure 10 explores the effects of varying  $g_b$ within the canopy Three depth dependences of  $g_b$  are used  $g_b$  is assumed to be invariant with depth ( $g_b = 0.04 \text{ m s}^{-1}$ ),  $g_b$  varies linearly with depth, and  $g_b$  varies exponentially with depth, more or less as in the Simple Biosphere Model (SiB) of Sellers et al (1986) In all three cases, the integral of  $g_b$  over the depth of the canopy is the same,  $0.32 \text{ m s}^{-1}$  It appears that variations in the profile form of  $g_b$  have some influence on the estimate of canopy-integrated conductance  $g_c$  and  $E_c$  (In

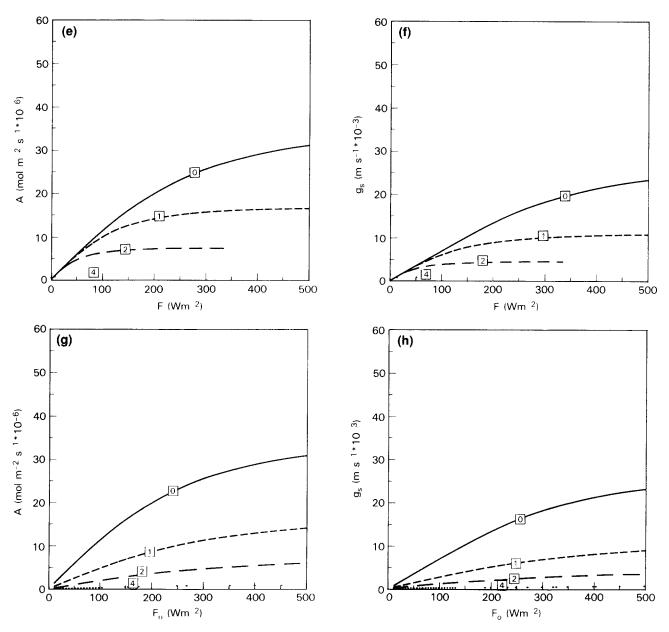


Figure 6 (continued) e,f) Same as a) and b) except  $\theta$ ,  $\beta = 0.8$  The top leaf", L = 0, responses were used to fit Eq. (5) and derive values of  $a_1$ ,  $b_1$ ,  $a_2$ ,  $b_2$ , and  $c_2$ , see Table 1 g,h) Same as c) and d) except  $\theta$ ,  $\beta = 0.8$ 

these figures and all subsequent equations, the use of a capital subscript refers to a bulk or canopy-integrated quantity)

The solutions shown in Figures 7–10 for the numerical integration scheme of Figure 5b were used to evaluate the accuracy of a simpler bulk integration scheme suitable for operational applications

Figure 5c shows the bulk integration scheme for the canopy Essentially, bulk or integral values of  $C_i$ ,  $g_s$ , and  $g_b$  ( $C_i$ ,  $g_c$ ,  $g_B$ ) are specified which in turn implies bulk values of  $C_s$ ,  $h_s$ ,  $A_s$ , and E The canopy transfers of CO<sub>2</sub> and H<sub>2</sub>O are thus treated as bulk integrated fluxes as in SiB It is further assumed that  $\mu \approx \overline{\mu}$ , so that f(L) is replaced by  $\overline{f(L)}$  in Eq (27b) The PAR and  $V_{\text{max}}$  extinction terms are now identical [both are defined by  $\overline{f(L)}$ ] so the canopy-depth portion of the integral can be separated from the physiological portion of the equation set Equation (27) can then be substituted into Eq (14a) and solved to give

$$w_{p} = \left[\frac{(a_{0} + b_{0}) - \sqrt{(a_{0} + b_{0})^{2} - 4\theta a_{0}b_{0}}}{2\theta}\right]\overline{f(L)} \qquad (28)$$

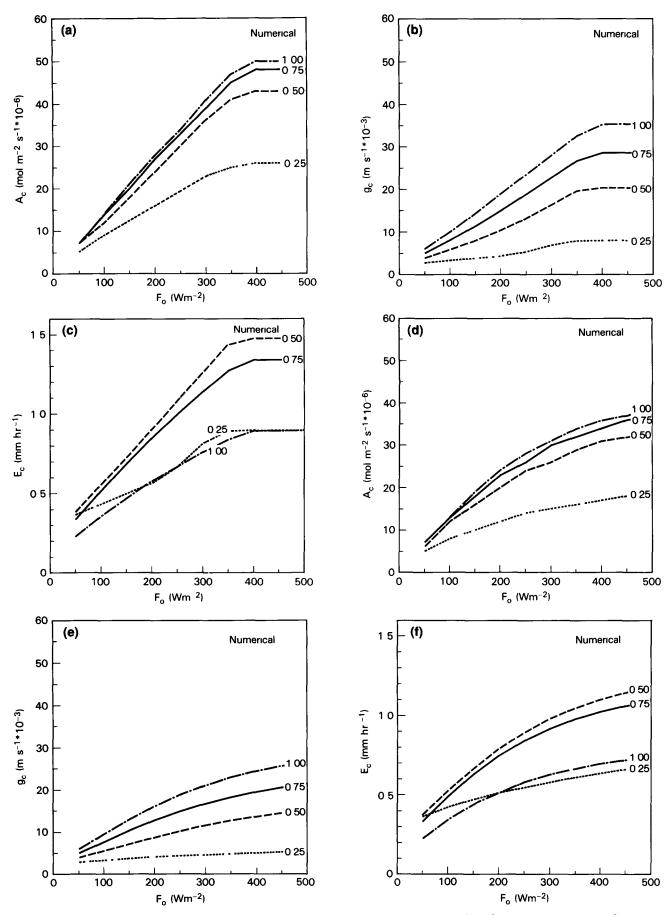


Figure 7 Canopy  $(L_r = 8 \ 0)$  photosynthesis  $A_c$ , conductance  $g_c$ , and transpiration  $E_c$  plotted against PAR  $(\mu = 0 \ 5)$  for different values of canopy air space relative humidity  $(h_a)$ ,  $h_a$  is marked on each curve Numerical method a)  $A_c$ , b)  $g_c$ , c)  $E_c$  for  $\theta$ ,  $\beta = 1$ , d)  $A_c$ , e)  $g_c$ , f)  $E_c$  for  $\theta$ ,  $\beta = 0 \ 8$ 

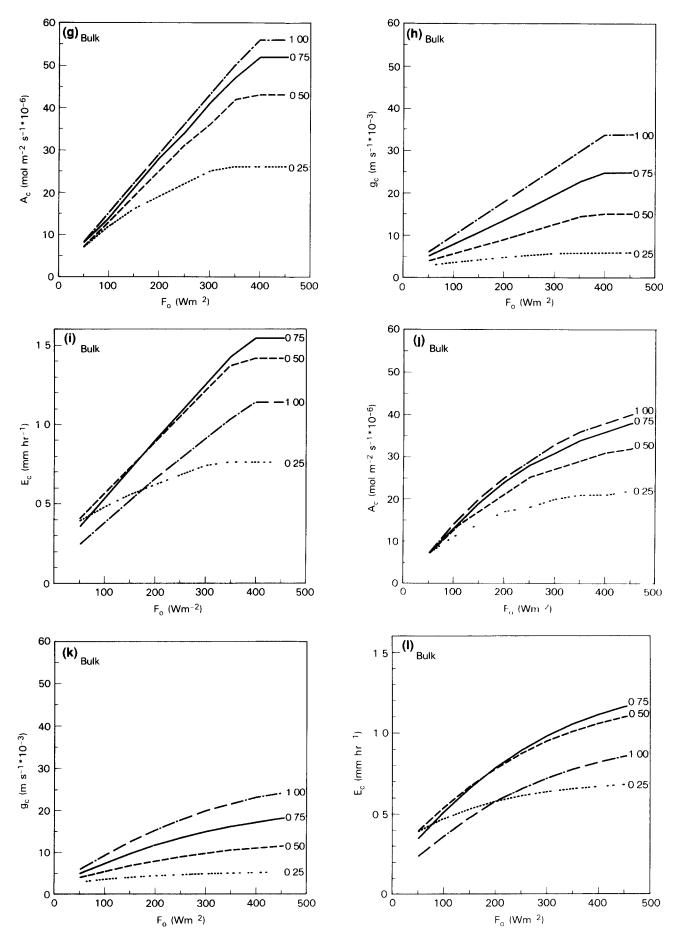


Figure 7 (continued) Bulk method g)  $A_{\epsilon}$ , h)  $g_{\epsilon}$ , 1)  $E_{\epsilon}$  for  $\theta$ ,  $\beta = 1$ , j)  $A_{\epsilon}$ , k)  $g_{\epsilon}$ , l)  $E_{\epsilon}$  for  $\theta$ ,  $\beta = 0.8$ 

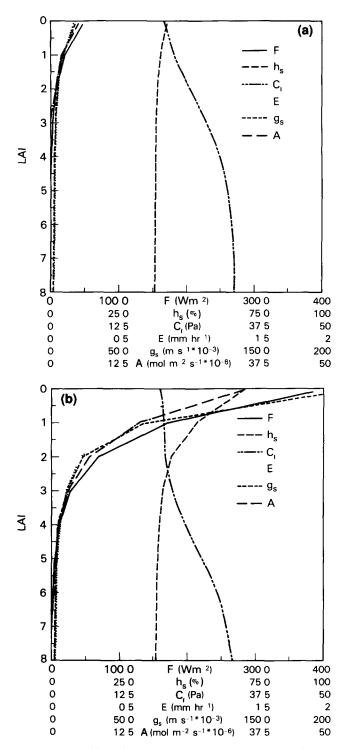


Figure 8 Profiles of PAR flux, F, leaf surface relative humidity  $h_i$ , leaf internal CO<sub>2</sub> concentration C<sub>i</sub>, leaf transpiration E, leaf conductance  $g_s$ , leaf assimilation A  $\theta$ ,  $\beta = 1$  0,  $h_a = 0.5$ ,  $\mu = 0.5$ ,  $R_d = 0$  all other parameters as in Table 2 a)  $F_0 = 50$  W m<sup>-2</sup>, b)  $F_0 = 400$  W m<sup>-2</sup>

The use of the bulk values  $C_I$ ,  $h_s$ , and  $C_s$  allows us to integrate (28) over the depth of the canopy to obtain  $w_P$  simply by integrating the f(L) term

$$w_{P} = \left[\frac{(a_{0} + b_{0}) - \sqrt{(a_{0} + b_{0})^{2} - 4\theta a_{0}b_{0}}}{2\theta}\right] \\ \times \int_{0}^{L_{T}} \overline{f(L)} \ dL$$
(29)

In the simple case of a homogeneous canopy, we can substitute Eq (25) into (29) to yield

$$w_{P} = \left[\frac{(a_{0} + b_{0}) - \sqrt{(a_{0} + b_{0})^{2} - 4\theta a_{0}b_{0}}}{2\theta}\right] \\ \times \left[\frac{1 - e^{-\bar{k}L_{T}}}{\bar{k}}\right]$$
(30)

Note that the second term in parenthesis can be written as

$$\Pi = \int_{0}^{L_{T}} \overline{f(L)} \ dL = \left[\frac{1 - e^{-\overline{k}L_{T}}}{\overline{k}}\right] \simeq \frac{\overline{FPAR}}{\overline{k}} \quad (31)$$

In Eq (30), the first term in parenthesis is simply the photosynthetic rate of the "top" leaves in the canopy, that is, those with the highest photosynthetic capacity,  $V_{\max 0}$ , exposed to the highest time-mean PAR flux,  $\overline{F}_0$ , and subject to  $C_I$ ,  $h_s$ , and  $C_s$  The second term, defined as  $\Pi$  in (31), acts as a simple scaling-up factor to relate the "top" leaf performance to canopy performance

The canopy equivalent of  $w_s$ ,  $w_s$ , is given by rearranging Eq. (12) to give

$$w_{s} = \int_{0}^{L_{T}} \frac{V_{m} dL}{2}$$
$$= \frac{V_{m0}}{2} \int_{0}^{L_{T}} \overline{f(L)} dL \qquad (32)$$

 $w_s$  and  $w_r$  can then be inserted into (14b) to determine  $A_c$ , the canopy gross assimilation rate The canopy respiration rate can be given by

$$R_{D} = \int_{0}^{L_{T}} R_{d} \quad dL = R_{d_{0}} \int_{0}^{L_{t}} \overline{f(L)} \quad dL$$
(33)

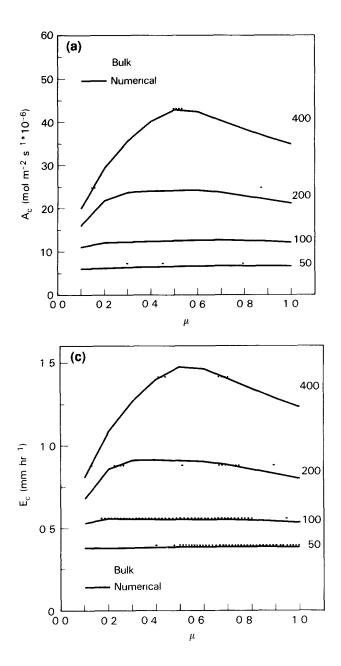
Equations (30), (32), and (33) can be combined to give the net canopy assimilation rate  $A_N$  As the integral of  $\overline{f(L)}$  occurs in all three equations,  $A_N$ can be written as

$$A_N = A_{n_0} \cdot \Pi, \tag{34}$$

$$A_{n_0} = f(a_0, b_0, V_{m_0}, R_{d_0})$$

where  $\Pi$  is given by Eq. (31)

 $A_{n0}$  is effectively the "single leaf" solution to Eqs (11)-(15) where constants appropriate to the



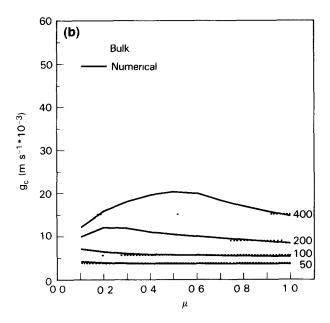


Figure 9 Dependence of canopy photosynthesis  $A_c$ , conductance  $g_c$ , and transpiration  $E_c$ , on the zenith angle  $\mu$  and intensity of the above canopy PAR flux,  $F_0$  Solid lines denote the numerical solution, dotted lines the bulk solution (The bulk solution shows no variation with  $\mu$ ) Numbers on lines refer to  $F_0$  Note that the combination of low  $\mu$  values and high  $F_0$  values (i.e., the areas on the figures to the left of the numerical solution maxima) are physically inplausible ( $\theta$ ,  $\beta = 1.0$ ,  $h_a = 0.5$ ,  $L_r = 8.0$ ,  $R_d = 0.0$ ) a)  $A_c$ , b)  $g_c$ , c)  $E_c$ 

"top" leaves are used,  $\Pi$  is the canopy-integral term which is dependent not only on FPAR but also on how the PAR is absorbed through the canopy, that is, canopy architecture, as specified by  $\bar{k}$ 

The combined canopy model is then completed with an integral form of the conductance equation (17)

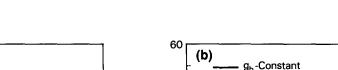
$$g_{\epsilon} = \frac{mA_{N}}{C_{S}}h_{s}p + bL_{\tau}, \qquad (35)$$

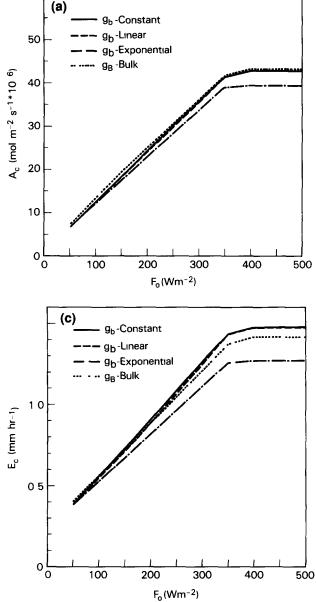
where  $A_N$ ,  $h_s$ , and  $C_s$  are bulk canopy values. In

Eq. (35), it is assumed that all leaves have the same "leakage" conductance b when  $A_N = 0$ 

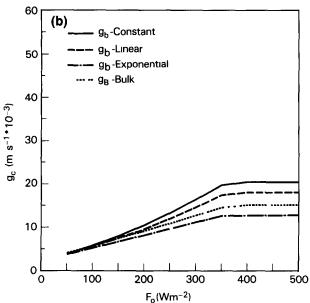
The system of equations corresponding to Figure 5c now consists of (27), (29), (31), (32), and (33) [summarized in (34)] and (35) These can be solved in exactly the same way as for a single leaf (see Fig 4)

The assumptions and simplifications involved in going from the system shown in Figure 5b to that in Figure 5c are nontrivial,  $\mu$  is set equal to  $\overline{\mu}$  and the variables  $C_{i}$ ,  $h_{s}$ ,  $C_{s}$ ,  $g_{s}$ , and  $g_{b}$  are all replaced by bulk canopy values. It is therefore





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Figure 10 Effects of the variation of  $g_b$  with canopy depth on values of  $A_c$ ,  $g_c$ , and  $E_c$  as a function of PAR  $(\theta, \beta = 1, 0, h_a = 0.5, \mu = 0.5, L_T = 8.0, R_d = 0.0)$  a)  $A_c$ , b)  $g_c$ , c)  $E_c$ Numerical solutions (----)  $g_b = 0.04$  m s<sup>-1</sup> (constant), (----)  $g_b = 0.06 - 0.04L/L_T$  (linear decrease) m s<sup>-1</sup>, (-----)  $g_b = 0.0925e^{-2L/L_L}$  (exponential decrease) m s<sup>-1</sup>. The integral of  $g_b$  over the depth of the canopy is 0.32 m s<sup>-1</sup> (=  $g_b$ ) in all three cases Bulk method (------)  $g_b = 0.025e^{-2L/L_L}$ 

interesting to explore the differences between the solutions given by the two methods

Figures 7g-l show the bulk method estimates of  $A_c$ ,  $g_c$  and  $E_c$  for the optimal case  $\overline{\mu} = \mu$ , the solutions are always within a few percent of the numerical scheme solutions, see Figures 7a-f

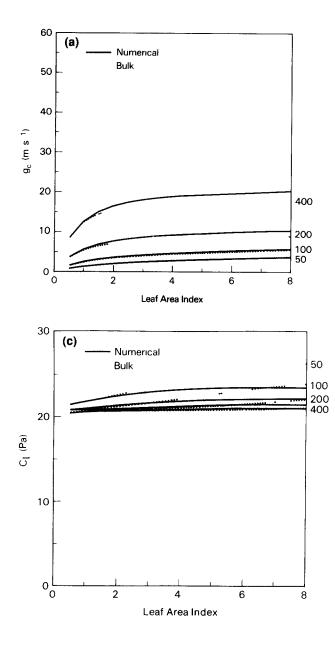
Figures 9 and 10 compare the values of  $A_c$ ,  $g_c$ , and  $E_c$  as given by the two methods over a range of PAR vectors and for a range of  $g_b$  profiles Except for the extreme and physically implausible case of low values of  $\mu$  combined with high values

of  $F_0$ , the bulk method yields fluxes that are close to those of the numerical scheme

Figure 11 compares the values of  $g_c$   $h_s$ , and  $C_i$  as given by the two integration schemes Weighted estimates of  $g_c$ ,  $h_s$ , and  $C_i$  for the numerical scheme are given by

$$g_c = \frac{1}{L_{\tau}} \int_0^{L_{\tau}} g_s \, dL,$$
 (36a)

$$h_s = \frac{1}{L_r} \int_0^{L_r} h_s \, dL,$$
 (36b)



$$C_t = \frac{1}{A_c L_T} \int_0^{L_T} A C_t \ dL \tag{36c}$$

The bulk model yields integrated estimates of these state variables that are close to those calculated with the numerical scheme

## **RELATING CANOPY BIOPHYSICAL PROPERTIES TO SPECTRAL REFLECTANCES**

Equations (34) and (35) appear to be effective canopy-integral forms of the leaf or chloroplast

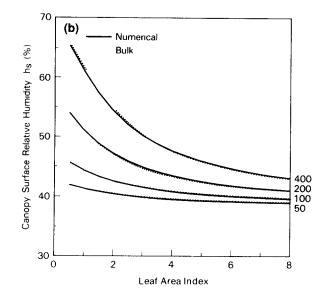


Figure 11 Comparison of bulk canopy conductance values,  $g_c$ , leaf surface specific humidity  $h_s$ , and internal CO<sub>2</sub> concentration  $C_i$ , as given by the numerical scheme, Figure 5b, and the bulk scheme, Figure 5c Estimates of  $g_c$  and  $C_i$  for the numerical scheme are given by Eq. (36) a)  $g_c$  comparison numbers denote PAR fluxes in W m<sup>-2</sup>, b)  $h_s$  comparisons symbols same as in a), c)  $C_i$  comparisons, symbols same as in a) ( $\theta$ ,  $\beta = 1.0$ ,  $h_a = 0.5$ ,  $\mu = 0.5$ )

level formulations of Farquhar et al (1980) and Collatz et al (1991) The nonlinear effects induced by varying solar angles and the profile gradients of governing state variables ( $C_i$ ,  $h_s$ ,  $g_b$ , etc.) within the canopy do not prevent the bulk integral method from providing useful estimates of the fluxes of CO<sub>2</sub> and H<sub>2</sub>O under normal conditions

The inclusion of these more sophisticated physiological models into the analysis leads to different canopy biophysical-spectral reflectance relationships than those described by Sellers (1987)

We can rewrite the canopy-integral equations

for the rate limiting processes governing photosynthesis as follows

$$w_c = v_{m_0} \Pi B_1, \tag{37a}$$

$$w_E = F_0 \quad \Pi B_2, \tag{37b}$$

$$w_s = V_{m_0} \Pi B_3, \qquad (37c)$$

$$R_D = V_{m_0} \Pi B_4, \tag{37d}$$

$$A_{c} = A_{n_{0}} \Pi B_{5} = f(w_{c}, w_{E}, w_{S}) - R_{D}, \qquad (37e)$$

$$g_c \approx A_{n_0} \prod B_6, \tag{37f}$$

where

$$B_{1} = \left[\frac{C_{I} - \Gamma^{*}}{C_{I} + K_{c}[1 + O_{2} / K_{c}]}\right],$$

$$B_{2} = \frac{G(\overline{\mu})}{\overline{\mu}}(1 - \omega_{v})\varepsilon\left[\frac{C_{I} - \Gamma^{*}}{C_{I} + 2\Gamma^{*}}\right],$$

$$B_{3} = 0 5,$$

$$B_{4} = 0 015,$$

$$B_{5} = 1 0,$$

$$B_{6} = \frac{mh_{s}p}{C_{v}}$$

In Eq (37), the canopy biophysical variables (left-hand side) are calculated as the product of three parameters or forcing variables (right-hand side) These parameters/forcing variables can be described as follows

## First Variable: Plant Physiology or Radiation Rate Limit Variable $(V_{m_0}, F_0, A_{n_0})$

 $V_{m_0}$  or  $F_0$  are the rate limiting factors governing the canopy response at saturating PAR fluxes and less than saturating PAR fluxes respectively  $A_{n_0}$ , appearing in (37e) and (37f), is a direct function of these two variables Previous sections have discussed the functional dependence of  $V_{m_0}$  on  $\mathbf{F}_0$  **n** The time history of  $\mathbf{F}_0 \cdot \mathbf{n}$  can be obtained from satellite climatological studies, for example, Froun and Gautier (1990) demonstrated how Geostationary Operational Environmental Satellite (GOES) data could be used to compute the diurnally varying PAR flux over a Kansas grassland area (the FIFE site) with a surface resolution of around 1 km<sup>2</sup> Such data could be composited to provide a global PAR climatology from which seasonally-varying, time-mean fields of  $\mathbf{F}_0 \cdot \mathbf{n}$  could

be produced The dependence of  $A_c$  and  $g_c$  on  $F_0$ is obvious

$$\frac{\partial A_c}{\partial F_0}, \frac{\partial g_c}{\partial F_0} = \Pi B_5, \Pi B_6, \quad \text{for } F_0 < V_{m_0} \frac{B_1}{B_2}$$
(38a)

= 0, for 
$$F_0 > V_{m_0} \frac{B_1}{B_2}$$
 (38b)

Equation (38) indicates that for nonsaturating PAR fluxes, the unstressed canopy photosynthetic rate and conductance respond almost linearly with changes in PAR with a slope that is directly proportional to  $\Pi$  and FPAR The effects of environmental stress or forcing are contained within the  $B_5$  and  $B_6$  terms Equation (38) holds for  $\theta$ ,  $\beta \rightarrow 1$ , lower values of  $\theta$  and  $\beta$  will give a gradual transition from (38a) to (38b) with increasing  $F_0$ 

Equation (38) and the supporting analysis indicate that all the leaves in the canopy saturate at the same value of  $F_0 = V_{m0}(B_1/B_2)$  This is because  $V_m$  is scaled according to the time-mean profile of PAR within the canopy

## Second Variable: Canopy PAR Use Parameter (II)

This parameter corresponds to  $\overline{\text{FPAR}}$  divided by the PAR extinction parameter  $\overline{k}$  It is the scaling parameter that relates canopy performance to the performance of the "top" leaves Typically,  $\overline{k}$  varies between 0.4 and 1, so that a continuous fully developed canopy ( $\overline{\text{FPAR}} \approx 1$ ) will perform at between one and three times the rate defined by the ensemble of "top" leaves

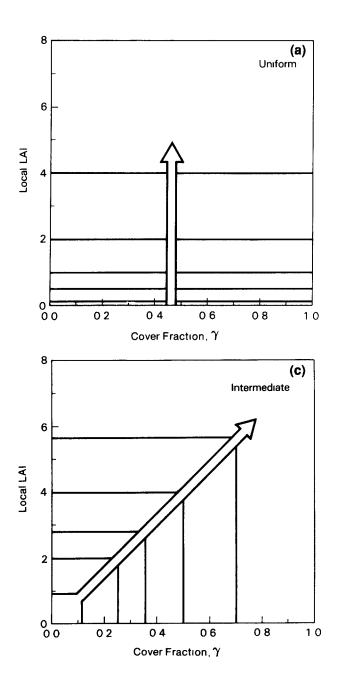
FPAR is the vegetation parameter most readily amenable to remote sensing [see Asrar et al (1984), Tucker et al (1981), Sellers (1985, 1987), Hall et al (1990), and papers in Asrar (1990)]. From this analysis, it appears that the value of FPAR associated with the radiationweighted time-mean PAR flux vector  $\overline{F_0}$  [see Eq (25)] combined with the corresponding value of  $\overline{k}$ , is the most useful canopy parameter for biophysical calculations. This implies that multiangle data should be acquired over vegetation canopies at solar angles corresponding to  $\overline{\mu}$  to obtain estimates of FPAR and  $\overline{k}$ 

If we neglect second-order feedback effects (see next section), all of the canopy biophysical rates are linear in  $\Pi$  [see Eqs (37) and (38)]

This represents a distinct improvement over the analysis of Sellers (1985) as now the contribution of canopy density and morphology ( $\Pi$ ) is cleanly separated from those of leaf physiology and radiation flux ( $V_{m0}$ ,  $F_0$ ) Sellers (1987) showed how under ideal conditions the simple ratio vegetation index, SR, was linearly related to FPAR provided that Eq (3) holds for the vegetation / type viewing sensor combination and the soil background is fairly dark. The chain of relationships

$$g_c, A_c \propto \Pi \propto \text{FPAR} \propto \text{SR}$$
 (39a)

or, simply,



$$A_{\epsilon} = f_{4}(SR), \qquad (39b)$$

$$g_{\epsilon} = f_{g}(SR), \qquad (39c)$$

where  $f_g$  and  $f_4$  are near-linear functions when  $\mu \rightarrow \overline{\mu}$ , indicates that SR or other SVI images may be used to calculate fields of  $A_{\epsilon}$ ,  $g_{\epsilon}$ , and  $E_{\epsilon}$  using a simple linear transform of the image combined with environmental forcings ( $F_0$ ,  $H_a$ ,  $T_a$ ,  $C_a$ , soil moisture stress, etc.) and some knowledge of the leaf physiology ( $V_{m_0}$ , m, etc.) In grasslands or areas with drought-deciduous vegetation, it is reasonable to assume that chronic soil moisture stress will be expressed as a decrease in  $\Pi$  so that direct

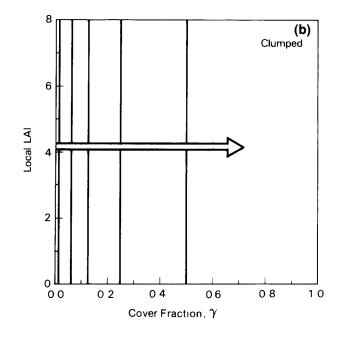


Figure 12 Local area index versus cover relationships for increasing area-averaged LAI,  $\langle L \rangle$  Arrow denotes increasing  $\langle L \rangle$  The heavy lines in each figure denote areaaveraged leaf area index values,  $\langle L \rangle$  of 0 1, 0 5, 1 0, 2 0, 4 0, and 8 0, increasing in the direction of the arrows Values of  $\langle L \rangle$  are equivalent to integrals under the heavy lines or the product of Cover Fraction and Local LAI a) Uniform (plane-parallel) canopy, b) clumped canopy constant local LAI,  $L_{max} = 8$ , cover fraction  $\gamma$ , varies with  $\langle L \rangle$ , c) intermediate case,  $L_{max} = 8\gamma$  These canopy configurations are used in Figure 13

knowledge of the soil moisture content might not be essential

In nature, variations in area-averaged leaf area index can occur in two ways First, a horizontally uniform canopy may vary in thickness (vertical dimension) This was the case studied by Sellers (1987), which led to the result that FPAR would be linearly related to SR (see Fig 12a) A second way is for the canopy to vary in area-averaged cover fraction This is often the case for coniferous trees or desert shrubs where the leaf area of an individual tree remains relatively constant, but plant abundance varies depending on ecological conditions (see Fig 12b) Hall et al (1990) used this second scenario to explore the utility of spectral second derivatives and SR data to determine FPAR, once again, these indices were more or less linear with FPAR A simple analysis is presented below which demonstrates that while the relationship between area-averaged leaf area index and FPAR (and therefore spectral vegetation indices, SVI) will vary depending on the horizontal and vertical distribution of the vegetation, the relationship between FPAR, SVI and  $A_c$ ,  $g_c$  remains invariant

Up to now, we have considered a plane parallel canopy of leaf area index  $L_T$  Let us now consider a landscape made up of identical homogeneous vegetation units, rather like box hedges, each with a *local* leaf area index of  $L_{max}$  and covering a total fraction  $\gamma$  of the landscape (or instrument field of view) Thus,

$$\langle \text{FPAR} \rangle = \gamma (1 - e^{-\bar{k}L_{\text{max}}})$$
 (40a)

The area-averaged leaf area index is given by

$$\langle L_r \rangle = \gamma L_{\max},$$
 (40b)

where the  $\langle \rangle$  symbols denote "area average"

Figure 13 shows the relationship between FPAR and  $\langle L_r \rangle$  for different degrees of "clumping" (values of  $\gamma$ ) Clearly, as  $\gamma$  decreases,  $\langle L_r \rangle$  must increase to maintain the same value of FPAR

However, an inspection of Eq. (37) and the supporting analysis shows that  $A_c$  and  $g_c$  are dependent on  $\Pi$ , that is, on FPAR/ $\overline{k}$ , and not on the spatial distribution of the vegetation density, provided that one ignores variations in the forcing/feedback terms  $B_1-B_6$  If we look at a vegetated region of area S, containing clumps of vegetation of varying size and density, but all having the same

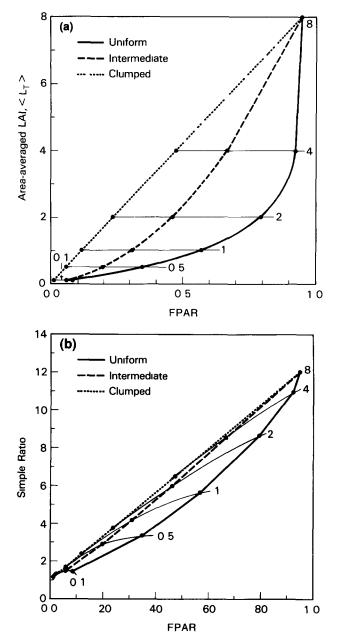


Figure 13 Variation of a) area-averaged total leaf area index  $\langle L_r \rangle$  with FPAR, and b) simple ratio vegetation index with FPAR for different levels of "clumpiness" parameters from Table 1 In a) and b), the thin lines ending in numbers refer to values of area-averaged leaf area index,  $\langle L_r \rangle$  (—) Spatially uniform canopy,  $\gamma = 1$   $L_r$  varies from 0 to 8, ( ) clumped canopy,  $\gamma = 0$  to 1,  $L_{max} = 8$ , (---) intermediate case,  $L_{max} = 8\gamma$  Refer also to Fig 12

baseline physiology  $(V_{\text{max0}})$  and leaf geometric/spectral properties  $(\bar{k}, \omega_c)$  overlying a soil background of uniform reflectance, we can write

$$\langle A_c, g_c \rangle = \frac{1}{S} \int_0^s A_c, g_c \ ds \tag{41a}$$

$$\propto \frac{1}{S} \int_{0}^{s} \Pi$$
, SVI  $ds$ , (41b)

where SVI, the spectral vegetation index, is SR or a spectral second derivative index (see Hall et al, 1990)

Now for a linear function, the area-integral and spatial average (multipled by the area) operators are equivalent

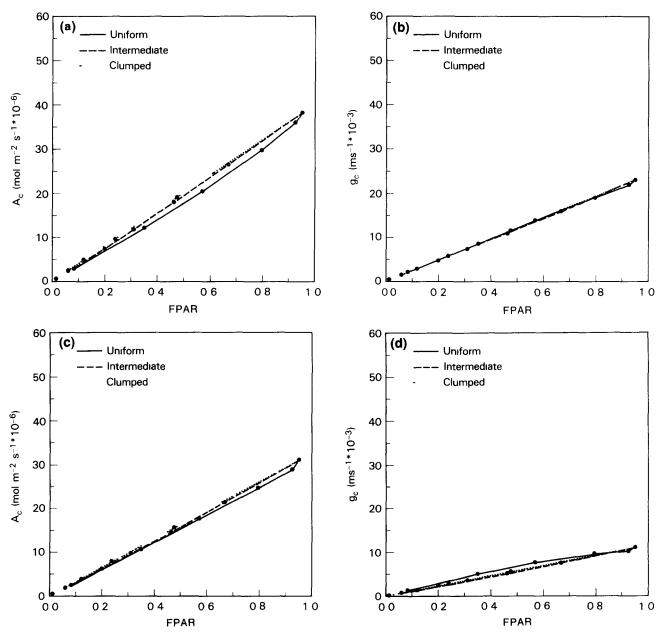
$$\langle \rangle \equiv \frac{1}{S} \int_{0}^{s} ds \qquad (42)$$

Since the functions relating  $A_c$ ,  $g_c$  and  $\Pi$ , SVI are linear or nearly so, we can rewrite (41) as

$$\langle A_c, g_c \rangle = \frac{1}{S} \int_0^s A_c, g_c \ ds \propto \langle \Pi \rangle \propto \langle \text{SVI} \rangle$$
 (43)

Equation (43) represents a powerful and

Figure 14 Variation of canopy photosynthesis  $A_c$  and conductance  $g_c$  with FPAR as calculated using the coupled stomatal-photosynthesis model (bulk scheme) shown in Figure 5c Different spatial distributions of  $\langle L \rangle$  are used corresponding to those shown in Figure 12 Dots denote values of  $\langle L \rangle = 01, 05, 10, 20, 40, 80$  (\_\_\_\_\_) Spatially uniform canopy,  $\gamma = 1, L_\tau$  varies from 0 to 8 (\_\_\_\_) clumped canopy,  $\gamma = 0$  to 1,  $L_{max} = 8, (--)$  Intermediate case,  $L_{max} = 8\gamma$  a b)  $A_c$  and  $g_c$ ,  $h_a = 1, c, d$ )  $A_c$  and  $g_c$ ,  $h_a = 0.5$  ( $\theta$ ,  $\beta = 0.8, F_0 = 400$  W m<sup>-2</sup>,  $\mu = 0.5$ )



counterintuitive result. The mean value of SVI as measured over an area should vary linearly with the areal integral and/or mean values of  $\Pi$ ,  $A_c$ , and  $g_c$ , to first order In areas with very high local values of  $L_{\tau}$ , higher-order effects (feedbacks between the vegetation and its immediate environmental through  $C_s$ ,  $h_s$ , etc.) may act to vary the coefficients  $B_1$ - $B_6$  and cause some local distortion of this relationship However, it is interesting that the inclusion of the ecophysiological optimality assumption of Eq. (21) into the equation set maintains the linearity of the SVI versus  $\Pi$  $A_c$ ,  $g_c$  relationship over a wide range of spatial scales and over heterogeneous vegetation density distributions (see Fig 14) This is a very different result from that obtained by Sellers (1985), who assumed invariant leaf physiology within the canopy, that is,  $V_{max} = V_{max0}$  for all L, and therefore derived a range of SVI versus  $A_c$ ,  $g_c$  relationships depending on the value of  $\gamma$  (see Fig. 15)

## Third Variable: Environmental Forcing or Feedback Term (B<sub>i</sub>)

For a given set of environmental conditions, the variables  $B_3$ ,  $B_4$ , and  $B_5$  can be effectively considered as constants over the area of integration, S. The variables  $B_1$ ,  $B_2$ , and  $B_6$  are the result of interactions between the vegetation biophysical process rates and the environmental forcings, the linkages being through  $C_r$ ,  $h_s$ , and  $C_s$ . These feedbacks are functionally the same as those for a single leaf (see Collatz et al, 1991).

#### SUMMARY

The leaf photosynthetic model of Farquhar et al (1980) and the leaf conductance model of Collatz et al (1991) can be analytically integrated over the depth of a vegetation canopy provided some simplifying assumptions are made. In practice, the bulk analytical canopy model yields values of net canopy assimilation rate  $A_c$ , canopy conductance  $g_c$ , and canopy transpiration  $E_c$  that are close to those provided by an exact (numerical) integration of the leaf models for normal environmental conditions

In defining the properties of the model canopy, the arguments of ecophysiological optimality, as invoked by Farquhar (1989) to describe the

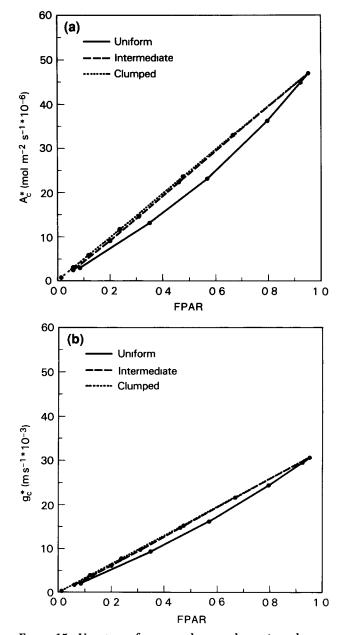
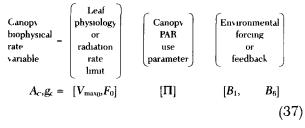


Figure 15 Variation of canopy photosynthesis  $A_c$  and conductance  $g_c$  with FPAR, as calculated with empirical leaf models in Eq. (5), following methods described in Sellers (1985) All symbols and conditions are the same as in Figures 14a and 14b a,b)  $A_c$  and  $g_c$ ,  $f(\Sigma) = 1$  (Canopy, soil properties from Table 1,  $F_0 = 400$  W m<sup>-2</sup>,  $\mu = 0.5$ ) Note that these values of  $A_c$  and  $g_c$  are higher than those shown in Figures 14a and 14b This is because *all* the leaves in the canopy have the same biophysical properties as the "top leaf", see Figures 1, 6c, and 6g Additionally,  $A_c$  and  $g_c$  are shown to vary markedly with vegetation heterogeneity using the methods of Sellers (1985) whereas, in the new formulation, they do not, compare with Figures 14a and 14b

profile of chloroplast Rubisco content within a leaf, were used to define the profile of leaf Rubisco content,  $V_{max}$ , within the canopy so that the re-

sulting profile of  $V_{\text{max}}$  follows the time-mean profile of PAR through the canopy

As a result, the contributions of leaf physiology, canopy density and geometry, and environmental forcing are separable in the integrated bulk canopy model, this was not the case for the "simpler" models used by Sellers (1985, 1987) This finding, Eq. (37), may be summarized by



This revised canopy model has the following interesting and useful properties

- i. Near-linear relationship between  $A_c$ ,  $g_c$  and incident PAR flux,  $F_0$ , for values of  $F_0$  less than saturation. All leaves within the canopy saturate at the same value of  $F_0$  due to the variation of  $V_{max}$  with depth.
- ii. Linear relationship between  $A_c$ ,  $g_c$  and the canopy PAR use parameter  $\Pi$ , where  $\Pi$  is equal to the radiation-weighted time-mean of canopy FPAR divided by the extinction coefficient for PAR,  $\overline{k}$

It should be possible to estimate  $\Pi$  by taking multiangle reflectance data over a target area for solar angles that correspond to the optimal timemean PAR flux ( $\mu = \overline{\mu}$ ) as defined by Eq. (26)

iii. Linear relationships between  $A_c$ ,  $g_c$ , II, and SVI simplify area-averaged biophysical calculations.

The SVI can be configured to yield a linear relationship with FPAR and hence  $\Pi$ , [see Eq (2)] This is true whether one considers a canopy varying in depth (Sellers, 1985, 1987) or areal cover fraction (Hall et al, 1990) or a heterogeneous combination of both Given such a condition (SVI $\propto$ FPAR,  $\Pi$ ) and an area containing vegetation of uniform physiology, leaf geometry, and spectral properties overlying a uniform background, the mean SVI for an area can be used to directly calculate the area integrals of the canopy photosynthetic rate  $A_c$  and conductance  $g_c$ 

$$\frac{1}{S}\int_{0}^{s} A_{c}, g_{c} \ ds = f_{A}(\langle SVI \rangle), F_{g}(\langle SVI \rangle), \quad (44)$$

where the functions  $f_A$ ,  $f_g$  are the same (nearlinear) functions relating canopy assimilation and conductance, respectively, to the SVI as those derived for a small-scale homogeneous vegetation cover (i e, a sample in the area) The angle brackets denote "area average"

The simplicity of these relationships should permit straightforward transformation of satellite imagery—time series of  $F_0$  from GOES data superposed on fields of SVI—to calculate regional fields of the (soil-moisture stress-free) canopy photosynthetic rates  $A_c$ , conductances  $g_c$ , and transpiration rates  $E_c$  The linearity of the SVI–II relationship should permit the use of coarse spatial resolution satellite imagery for this application

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